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THE ROYAL AERONAUTICAL SOCIETY

AUGUST 1960

CONTENTS

NOTICES

AN INTRODUCTION TO THE FLOW ABOUT PLANE SWEPT-BACK WINGS AT TRANSONIC SPEEDS — E. W. E. Rogers and I. M. Hall

SIR HUBERT WILKINS, FATHER OF ARCTIC AVIATION — J. Grierson

ENGINEERING ASPECTS IN MAN POWERED FLIGHT — B. S. Shenstone

MAN AS AN AERO ENGINE — D. R. Wilkie

SOME PROBLEMS OF POLAR MISSILE CONTROL — D. Best

TECHNICAL NOTES

The Initial Instability of an Elastic Sheet Reinforced by Stringers and Skew Ribs—G. G. Pope. On the Breakdown at High Incidences of the Leading Edge Vortices on Delta Wings—B. J. Elle. Initial Strains in the Matrix Force Method of Structural Analysis—J. H. Argyris and S. Kelsey.

THE BRANCHES

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VOLUME 64 AUGUST 1960 NUMBER 596

CONTENTS

		PAGE
Notices		XXXIX
Lecture Theatre Opening Ceremony—Second I.C.A.S. Congress, Zurich—Russian Translations—Associate Fellowship Examinations, December—Royal Aeronautical Society University Prizes—Elliott Memorial Prize—Honours Awarded to Members—President of the S.B.A.C.—Captain and Mrs. Pritchard: New Address—Birthday Honours, 1960—Supplement to Oxford English Dictionary—News of Members—Elections.		
An Introduction to the Flow about Plane Swept-back Wings at Transonic Speeds	E. W. E. ROGERS and I. M. HALL	449
Sir Hubert Wilkins, Father of Arctic Aviation	J. GRIERSON	465
Man Powered Aircraft Group		
Engineering Aspects in Man Powered Flight	B. S. SHENSTONE	471
Man as an Aero Engine	D. R. WILKIE	477
Astronautics and Guided Flight Section		
Some Problems of Polar Missile Control	D. BEST	482
Technical Notes		489
The Initial Instability of an Elastic Sheet Reinforced by Stringers and Skew Ribs—G. G. POPE. On the Breakdown at High Incidences of the Leading Edge Vortices on Delta Wings—B. J. ELLE. Initial Strains in the Matrix Force Method of Structural Analysis—J. H. ARGYRIS and S. KELSEY.		
The Branches		496
Graduates' and Students' Section		497
The Library		498
REVIEWS: The Theory of Thin Shells—The Beauty of Gliding—Great Aircraft—Laplace Transforms for Electronic Engineers—Symposium on Ground Effect Phenomena.		
ADDITIONS TO THE LIBRARY. REPORTS.		

All communications for publication in the JOURNAL *should be addressed to the Editor,* JOAN BRADBROOKE, *A.R.Ae.S.; all communications on general matters affecting the Society should be addressed to the Secretary,* A. M. BALLANTYNE, *T.D., B.Sc., Ph.D., F.R.Ae.S., Hon.F.C.A.I., F.I.A.S.*

None of the papers or paragraphs must be taken as expressing the opinion of the Council of the Royal Aeronautical Society unless otherwise stated.

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NOTICES

Second ICAS Congress—Zurich

Members wishing to attend the Second International Congress of the Aeronautical Sciences at Zurich, 12th-16th September, should inform the Secretary, 4 Hamilton Place, **immediately**. The programme was given in the July JOURNAL.

Russian Translations

Members who would be interested in translating Russian aeronautical papers into English are asked to write to the Secretary.

Honours Awarded to Members

Sir Frederick Handley Page, *Honorary Fellow* and Past President of the Society, was presented with the Ludwig-Prandtl Ring 1960, at Bonn on 5th July 1960.

President of the S.B.A.C.

Sir George Dowty, *Fellow* and Past President of the Society, has been elected President of the Society of British Aircraft Constructors, 1960-1961.

Birthday Honours 1960

Commander E. H. Banfield, R.N. (*Associate Fellow*) was awarded the O.B.E., and Flt. Lieut. J. Beattie (*Associate*) the M.B.E., in the Birthday Honours List.

Supplement to Oxford English Dictionary

The following is the eighth list of aeronautical terms for the Supplement to the Oxford English Dictionary for which assistance in tracing earlier references is required. If members know of an earlier use than that given for any word in the following list they are asked to write to the Editor, Oxford English Dictionary Supplement, 40 Walton Crescent, Oxford, giving the reference(s), date, author, title, chapter and page. The Editor wishes to thank those who have helped with previous lists.

course, sb. (route or direction of aircraft)	1945	T. A. Dickinson, *Aeronautical Dictionary*
crabpot valve	1918	W. E. Dommett, *Dictionary of Aircraft*
crash-land, v.	1941	P. Richey, *Fighter Pilot*
crash-landing, vbl. sb.	1935	*Journal of the Royal Aeronautical Society*, XXXIX
crash wagon	1938	*Time*, 5th September
crate (slang = aeroplane)	1928	*Daily Mail*, 7th May
crazy flying	1926	*Morning Advertiser*, 3rd May
crop dusting (from an aeroplane)	1920	L. Bairstow, *Applied Aerodynamics*
cruise, v. (of vehicles or aircraft)	1934	*Discovery*, December
decalage	1918	Webster, *New International Dictionary* (Addenda)
deck (colloq. = ground)	1941	*War Illustrated*, 29th August
dicey, dicy (Air Force slang)	1950	Nevil Shute, *A Town Like Alice*
dinghy (small inflatable boat)	1936	*Air Stories*, December
dispersal (point on airfield)	1940	*Flight*, 7th November
ditch, v. (make a forced landing on the sea)	1941	*Times Weekly*, 15th October
ditching, vbl. sb. (as above)	1943	*Life*, 29th November
dive-bomb, v.	1940	*The Times*, 23rd July
dive-bomber	1939	*The Times*, 29th September
dive-bombing, vbl. sb.	1935	*Evening News*, 11th July

Lecture Theatre Opening Ceremony

The Opening Ceremony for the Lecture Theatre will be on Friday 28th October. Details and application forms for tickets will be given in the September JOURNAL.

Associate Fellowship Examinations—December

The closing date for Candidates in the United Kingdom for December 1960 Examinations is 31st August. Entry forms may be obtained from the Secretary. The closing date for entries outside the U.K. was 30th June.

Royal Aeronautical Society—University Prizes

The following have been awarded Royal Aeronautical Society University Prizes for 1960.

Cambridge University	D. W. Price, St. Catharines College.
Southampton University	A. J. Pretlove.
Bristol University	J. Armitt
Imperial College	F. G. Ross, J. A. Holt } Joint Award
Belfast University	D. L. I. Kirkpatrick
College of Aeronautics	C. G. B. Mitchell

Elliott Memorial Prize

The Elliott Memorial Prize for the second term in 1960 has been awarded to Corporal Apprentice A. J. Phillips, and will be presented at the Halton Graduation Prizegiving.

Captain and Mrs. Pritchard

Captain and Mrs. Pritchard have moved from Preston Park to 34 Flanchford Rd., Reigate Heath, Reigate, Surrey.

News of Members

Sqn. Ldr. J. A. Beadle (*Associate Fellow*) formerly Engineer Officer at R.A.F. Scampton, has been posted to R.A.F. Germany, 2nd T.A.F.

W. R. Bendall (*Associate Fellow*) formerly Operational Liaison Engineer with Rolls-Royce is now Performance and Development Engineer with Middle East Airlines, and is at present at de Havilland Aircraft Ltd., during the construction of M.E.A.'s Comet Fleet.

D. A. Birkett (*Associate*) formerly in charge of Technical Advertising at Nevin D. Hirst (Advertising), has resigned from the Board and joined Gee Advertising Ltd.

W. K. Campe (*Associate Fellow*) formerly Head of the General Engineering Group, Signal Research and Development Establishment, Ministry of Supply, is now Development Project Officer, Ministry of Aviation, St. Giles Court.

C. D. Carmichael (*Associate Fellow*) formerly Assistant Chief Engineer, D. Napier and Son Ltd., Acton, is now Chief Engineer, Engine Components Division, Associated Engineering Ltd., Leamington Spa.

Dr. W. Cawood (*Fellow*) formerly Deputy Controller of Aircraft (Research and Development) has been appointed Chief Scientist to the War Office.

E. G. Collinson (*Associate Fellow*) formerly Chief Designer, British Messier Ltd., is now Chief Mechanical Engineer at Short Brothers and Harland Ltd., Belfast.

H. B. Cundall (*Associate Fellow*) formerly Development Engineer with Hunting-Clan Air Transport Ltd., and Airwork Ltd., has joined the Sales and Service Organisation of Rolls-Royce, Aero Engine Division.

W. H. Dudley (*Associate*) formerly Senior Engineer, Inertial Guidance Group, Smiths Aircraft Instruments has joined the Technical Sales staff of Vernon Industries Ltd., Kirby.

G. Esilman (*Graduate*) formerly Flight Test Engineer, Avro Aircraft Ltd., Malton, is now an Aerodynamicist at the de Havilland Aircraft Co. of Canada.

Commander G. J. FAIRLEY (*Associate Fellow*) formerly in the Office of Flag Officer, Air (Home), Lee-on-Solent, has been appointed Air Engineer Officer at the R.N. Air Station, Halfar, Malta.

W. FOSTER (*Associate Fellow*) formerly in the Aircraft Design Office, Blackburn (Dumbarton) Ltd., is now Senior Designer (Armaments), Blackburn Aircraft Ltd., Brough.

C. M. GALLEN (*Associate*) formerly an Aerodynamicist, Short Brothers and Harland Ltd., is now a Project Engineer, English Electric Aviation Ltd., Warton.

R. M. GIBSON (*Associate Fellow*) formerly Senior Draughtsman, Bristol Aeroplane Co., is now an Engineer "A", Airframe Structures Department, Canadair Ltd., Montreal.

Sqn. Ldr. W. B. GREEN (*Associate Fellow*) formerly Technical Branch, R.A.F., Ministry of Supply (G.W. Air), is now Engineer Officer, O.C.U., R.A.F., Chivenor.

S. J. HAMMOND (*Associate*) formerly Technical Officer, Defence Research Board, Canadian Armament Research and Development Establishment, Quebec, is now Production Officer (Missile Research and Development Group), Ottawa.

G. R. HAMPSHIRE (*Associate*) formerly Chief Engineer with Air Links, Gatwick, is now Chief Inspector, Aviation Division of the Pressed Steel Co.

Wg. Cdr. R. E. HAVERCROFT (*Associate*) formerly R.A.F. Support Unit, H.Q. Allied Air Forces, Central Europe, is now with the Department of the Chief of the Air Staff, Air Ministry, Whitehall.

Wg. Cdr. M. HERMISTON (*Associate Fellow*) formerly R.A.F. Waddington, is now at Bomber Command, R.A.F., High Wycombe.

J. O. HITCHCOCK (*Associate Fellow*) formerly Sales Director of Mond Nickel Co. Ltd. and a Director of Henry Wiggin and Co. Ltd., is now Managing Director of Mond Nickel and Deputy Chairman of Henry Wiggin.

A. D. HOWARTH (*Associate*) formerly in charge of the Technical Training Section is now Assistant Design Office Manager at Blackburn Aircraft Ltd., Brough.

J. C. KING (*Associate Fellow*) formerly Export and Administrative Engineer, English Electric Co. Ltd., London, is now at their Phoenix Works, Bradford.

G. MAGEE (*Associate*) formerly Deputy Inspector-in-Charge, A.I.D., Blackburn Aircraft Ltd., is now Ministry of Aviation Aircraft Division Representative, Salisbury, Australia.

J. MERRICK (*Associate Fellow*) formerly Engineer, Installation Dept., de Havilland Aircraft Co. Ltd., Hatfield, is now an Engine Installation Designer with Bristol Aircraft Ltd., Filton.

S. V. MOLONY (*Student*) formerly Cadet Engineer, Government Aircraft Factory, Victoria, is now Trials Results Engineer with the Government Aircraft Factory.

M. B. MORGAN (*Fellow*) formerly Scientific Adviser to the Air Ministry has been appointed Deputy Controller of Aircraft (Research and Development), Ministry of Aviation.

Flying Officer M. MURDEN (*Graduate*) formerly Engineer Officer at R.A.F. Bassingbourn, has been posted to No. 8 Squadron, Aden, as Engineer Officer.

Flt. Lt. D. C. D. POTTER (*Graduate*) formerly R.A.F., Driffield, Yorkshire, is now on Instructing Duties (ballistic missiles) R.A.F., Hemswell, Lincs.

K. RAMA RAO (*Graduate*) having completed the NATO Guided Missile Course at the College of Aeronautics, Cranfield, has returned to his former position of Junior Scientific Officer, Ministry of Defence, New Delhi.

E. RHOADES (*Associate Fellow*) formerly with the R.A.F., is now Sales Engineer with British Oxygen Aviation Services, Harlow.

G. G. ROBERTS (*Fellow*) has resigned from the Board of Smiths Aircraft Instruments and is now Technical Director, Cossor Radar and Electronics Ltd., Harlow.

P. T. ROSS (*Graduate*) formerly Development Engineer, Project and Development Branch, B.E.A., is now Research Officer with Air Research Bureau, Brussels.

Group Capt. E. SHIPLEY (*Associate Fellow*) formerly an Independent Consulting Engineer is now a Development Project Officer in the Ministry of Aviation.

D. E. SITCH (*Student*) formerly Stressman with de Havilland Propellers Ltd., is now a Technical Adviser with Hoffmann Manufacturing Co. Ltd., Chelmsford.

A. TAYLOR SMITH (*Graduate*) formerly with Avro Aircraft, Malton, has been assigned by IBM to Product Programming, San Jose, California, from IBM Western Region Headquarters, Los Angeles.

F. E. SPARKS (*Associate*), formerly in the Technical Branch, B.O.A.C., is now Technical Manager, Electrolube Ltd., London.

P. A. E. STEWART (*Associate*) formerly Development Engineer, Saunders-Roe Ltd., I.O.W. is now Rocket Engineer in the Astronautics Section of Hawker Siddeley Aviation, Advanced Projects Group.

P. H. STOKES (*Fellow*) formerly Technical Director of H. M. Hobson Ltd., is now Technical Adviser to Teddington Aircraft Controls Ltd.

ELECTIONS

The following is a list of elections and transfers of membership:—

Associate Fellows

Roy Frederick Burrows (*from Graduate*)
Arthur William Caswell
Gwyn William George Court
Kenneth Cubbage
David Bruce Elkman (*from Graduate*)
Dene Evans
Walter Lawrence Grant
Wallace William Hammond
Rowland Baron Harrison
Donald Edsall Hatton (*from Graduate*)
Stanley John Hubbard
Derek Stuart Kendall (*from Graduate*)
William Frank Maidment
Nelson William Maskell
Derek Henry Michel (*from Associate*)
Geoffrey Keith Charles Pardoe (*from Graduate*)
John Edward Phythian
Douglas Hector Norrie (*from Graduate*)
William Lockhart Rait
William Dobson Richmond
Thomas Henry Robjohn
L. Eugene Root
Peter Travers Ross (*from Graduate*)
John Rye (*from Graduate*)
Michael Frank Simpson (*from Graduate*)
Maurice Saxon Snell
Nallepilli Ramalinga Subramanian
Alexander Syme (*from Graduate*)
James Velkou (*from Graduate*)
William Gordon Watson (*from Graduate*)
Edward Clement Weston (*from Graduate*)
Stanley Williams

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William John Dunn
Paul Harold Taylor Farquharson
Eric Taylor Furness
Ronald Hurst
Ali Khan Mubarak
Mordechay Savitsky
Arthur Carlton Smith
Arthur Frederick Southgate
Rolf von Bahr
Philip Edmund Warcup

Graduates

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Leonard Bernstein (*from Student*)
Michael David Clarke (*from Student*)
Nigel Haughton (*from Student*)
Denzil Joseph Isaaks (*from Student*)
Noel John Seyb
Arthur James Smith
John Weeks (*from Student*)

Students

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Cyril Francis John Chresta
Clive Dugmore Clarke
Richard Francis Dunsmore
John Edward Hinton
Vashdev M. Lalla
Graham Marsh Leach
Anthony Valentine Awing Pantin
Kenneth Frederick Smy
Roger Steele
Abdul-Fattah Tuqan

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Ronald Stanley Anker-Simmons
David John Backhouse
Eric Frank Grove
Gillian Helen Mackay
Colin Edward Read
Allan Frederick Rich
Gerald Hercules Robinson
Bernardus Arthur Marie Schreiner
Eelco J. W. Schuller

The Journal of the Royal Aeronautical Society
INCORPORATING THE INSTITUTION OF AERONAUTICAL ENGINEERS AND THE HELICOPTER ASSOCIATION OF GREAT BRITAIN

VOLUME 64 AUGUST 1960 NUMBER 596

An Introduction to the Flow about Plane Swept-back Wings at Transonic Speeds*

by

E. W. E. ROGERS, D.I.C., M.Sc.(Eng.), A.F.R.Ae.S., and I. M. HALL, Ph.D., M.Sc.

(Aerodynamics Division, National Physical Laboratory)

SUMMARY: A brief survey is made of the way the flow develops about wings with leading edges swept at about 50° as the stream Mach number rises from a subsonic to a supersonic value. The shock pattern which occurs may be complex. Three aspects of the wing flow are discussed in more detail, including the possible conditions for shock-induced separation of the surface boundary layer. The effect of the changing flow pattern on the overall wing lift, drag and pitching moment is commented upon.

1. Introduction

The use of swept-back wings to avoid certain undesirable compressibility effects is now an accepted part of aircraft design. The potential benefits of sweepback were first pointed out by Busemann some 25 years ago and ten years later swept-back wings began to be used on high-performance military aircraft in order to take full advantage of the increased power available for the new turbo-jet and rocket engines.

At a very early stage a simple theory was put forward to allow estimates to be made of the aerodynamic performance of swept-back wings. In essence, this suggested that on an infinitely long wing the flow was dominated by the component of stream Mach number normal to the leading edge, *i.e.* $M_0 \cos \phi$, where ϕ is the sweepback angle and M_0 the stream Mach number. The Mach number component along the span of the wing ($M_0 \sin \phi$) is equivalent to a translation of the wing in the spanwise direction and was assumed to play no part in determining the shape of the chordwise pressure distribution. It thus follows that once the unswept, or two-dimensional, section characteristics are known, the behaviour of the wing at any sweepback can be predicted, provided the wing incidence, stream Mach number, and surface pressure coefficients are correctly represented. This correspondence has been demonstrated on several wind tunnel tests; results from recent N.P.L. experiments are shown in Fig. 1. The agreement between the three pressure distributions is very good despite the fact that the wings were not of infinite span.

The agreement deteriorates, however, as the stream Mach numbers increase and the components normal to the leading edges of the swept wings approach or exceed unity (Fig. 2). Shock waves may then exist on the swept wings at positions which are different from those predicted from a knowledge of the unswept wing behaviour and it is evident that the simple sweepback theory is no longer applicable to this particular case. One of the most important reasons for this failure is that the wings used were of finite span, so that the upstream and downstream junctions with the wind tunnel walls exerted a considerable influence over the wing flow, an influence which was very small at the lower stream speeds of Fig. 1, but which increased rapidly as the flows became transonic and supersonic.

Conditions at the junctions of the wing and the tunnel walls resemble in many respects those at the root and tip of a finite swept-back wing and hence similar departures from the simple yawed-wing concepts might be expected and indeed have been observed, particularly if the wing is of moderate or low aspect ratio. The flow patterns which result from transonic and supersonic flows about swept-back wings may in fact be quite complex and a detailed study of this field has only been undertaken in the past few years, even though aircraft with swept wings have been flying in transonic conditions for considerably longer.

This lag between research and practice is not unfamiliar and in the present case arose mainly because quite appreciable and acceptable delays in the onset of shock-wave drag could be obtained without the need for a detailed understanding of the flow. However, present-day designs (particularly of transport aircraft) demand the maximum performance from the wing and hence a more complete knowledge of the flows about simple swept-back wings. This knowledge can form a background against which the validity and possible success of various sophisticated techniques, such as fuselage contouring and wing warp, can be assessed.

The present paper is intended as a contribution to the understanding of transonic flows about swept wings. It is restricted to a discussion of work undertaken in the Aerodynamics Division of the National Physical Laboratory on plane wings having simple plan forms and leading edges swept at about 50°. In all cases a band of carborundum powder was put on the wing leading edges so that boundary layer transition took place at the roughness band. This means that the boundary layers on both surfaces of the models were turbulent. The effects of bodies on the wing flow will not be considered.

*A Lecture given before the Society on 1st December 1959.

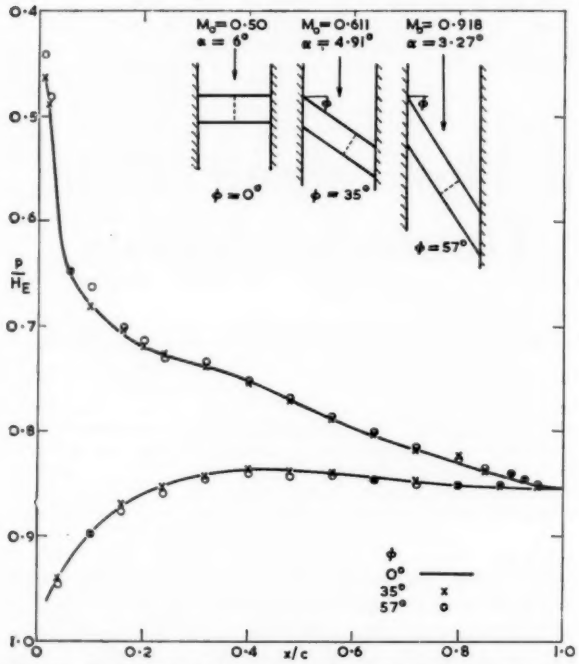


FIGURE 1. Validity of simple sweepback theory for component Mach number of 0·5 normal to leading edge. H_E is the effective total pressure of this component and p the local static pressure.

Moreover it has not been thought practicable to relate the N.P.L. work to that carried out elsewhere, but the authors wish to acknowledge the considerable value of many discussions held with members of the other Government establishments, aircraft firms and the Aircraft Research Association.

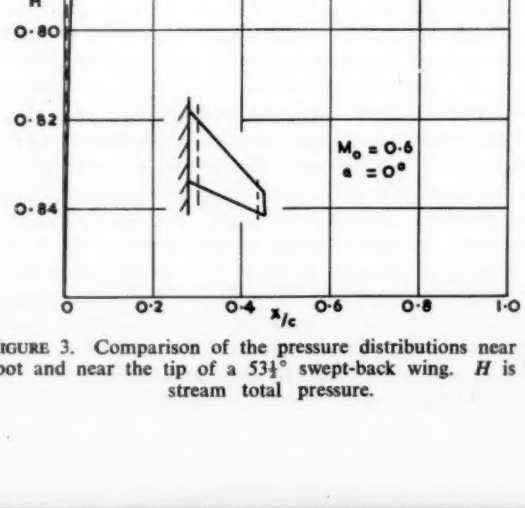


FIGURE 3. Comparison of the pressure distributions near the root and near the tip of a 53½° swept-back wing. H is the stream total pressure.

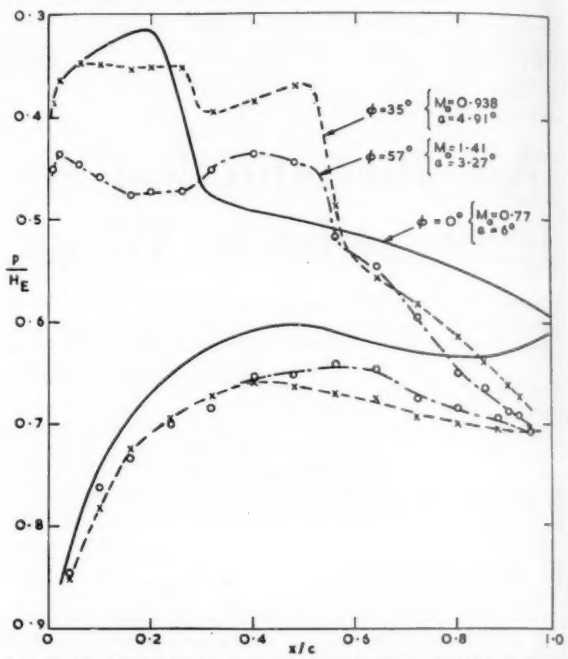


FIGURE 2. Deterioration of agreement shown in Fig. 1 when Mach number component normal to leading edge is raised from 0·50 to 0·77.

2. The Flow near Zero Lift

It is convenient to consider first the way in which the flow develops about a simple swept wing at or near zero lift as the Mach number is increased. The effects of applying moderate and large incidences will be described later.

The chordwise pressure distribution on a plane, finite swept-back wing varies across the span, so that for a pressure distribution of the roof-top type, for example, the maximum surface velocity is reached much farther aft at the root and farther forward at the tip compared with the basic infinite-wing distribution (Fig. 3). These distortions due to root and tip are of great importance and determine the subsequent development of the flow pattern. Physically the effects may be thought of as due to the influence on a given station of stations just inboard and outboard, and hence displaced slightly upstream and downstream. Because of this displacement a pressure gradient exists normal to the local flow direction on both sides of the chosen station. Its chordwise pressure distribution will therefore depend on the relative magnitude of the inboard and outboard influences. These will be equal and opposite on an infinite wing and the theoretical yawed-wing distribution should result.

On a finite wing, the outboard influence is strongest at the root, so that local pressures are increased over the front part of the section, but reduced over the rear. The converse applies towards the wing tip. This is equivalent to the lines of constant local velocity running obliquely across the wing at a somewhat lower sweep than the wing leading edge, and with the highest local velocities near the tip. This is sketched in Fig 4. The velocity contours close to the foot and tip curve sharply

in order to conform to the appropriate boundary conditions and may form complete loops on the wing surface. At the root the air is forced to flow in the free-stream direction, whereas farther outboard its direction is less constrained.

An increase in stream Mach number causes appropriate increases in the local surface velocities, and ultimately supersonic flow occurs close to the wing tip. The formation of shock waves on an infinite swept-back wing is usually held to correspond to the fact that the local Mach number normal to the velocity contours exceeds unity; this is by direct analogy with the two-dimensional case. It follows that on an infinite swept-back wing supersonic velocities may be attained locally with no penalty in wave drag except for that associated with the bow wave, which appears as soon as the stream flow becomes supersonic.

There are difficulties, however, in extending these ideas to the finite wing, particularly when the velocity contours may exist as closed loops and hence contain a section which is normal to the stream direction. On this argument, for contours of the type shown in Fig. 4, a shock wave should appear soon after the flow near the tip becomes supersonic and this does in fact seem to be the case in practice for the plane wings of simple plan form considered in this paper. Shock waves will thus occur much earlier than would be expected from the simple sweep analogy. For example, they just appear at zero incidence on a tapered swept-back wing with 53½° leading edge sweep and 6 per cent thick streamwise section at a stream Mach number of about 0·90. By comparison, two-dimensional tests on the same section (which will not have quite the same pressure distribution as the tip section of the finite wing) show that shock waves form at a Mach number of about 0·86.

This shock wave which appears on the finite wing has been called the initial tip shock. It lies almost normal to the plane of the wing and the stream direction, and the inboard spread is not very large as can be seen in Fig. 5. The initial tip shock moves rearward and extends slightly inboard as the stream Mach number increases (Fig. 6), but its development is restricted by the presence of a second shock (called the rear shock) which generally forms at about the same time as the initial tip shock. The rear shock is associated primarily with conditions at the wing root, though its influence extends across the whole of the wing span. Its origin may be conveniently considered in the following way.

If the effect of the wing thickness is to increase only the velocity component normal to the leading edge (this is again a use of simple sweepback concepts), leaving the spanwise component unchanged, the flow direction must then be inclined towards the root, as in Fig. 7. A simple relationship exists between the flow deflection θ and the local Mach number M_L for a given leading edge sweepback and stream Mach number. The streamlines over the wing surface will be curved therefore, as sketched in Fig. 8, but at the wing root the flow is constrained to follow the wall. The compression system required to achieve this propagates across the wing span and ultimately may coalesce near the tip to form a shock wave of finite strength (Figs. 8 and 9).

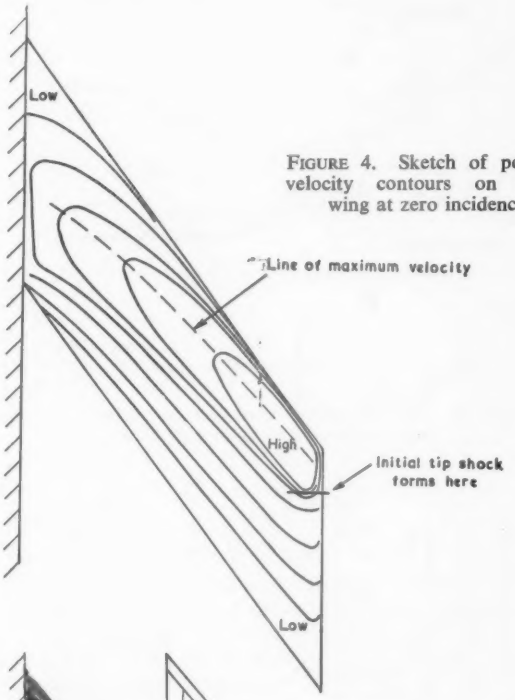


FIGURE 4. Sketch of possible velocity contours on swept wing at zero incidence.

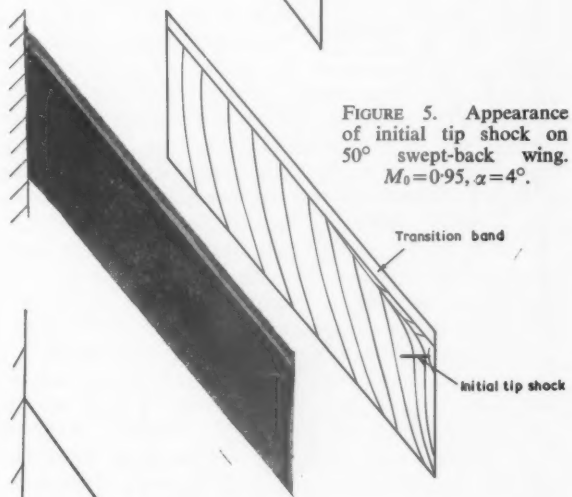


FIGURE 5. Appearance of initial tip shock on 50° swept-back wing. $M_0=0{\cdot}95, \alpha=4°$.

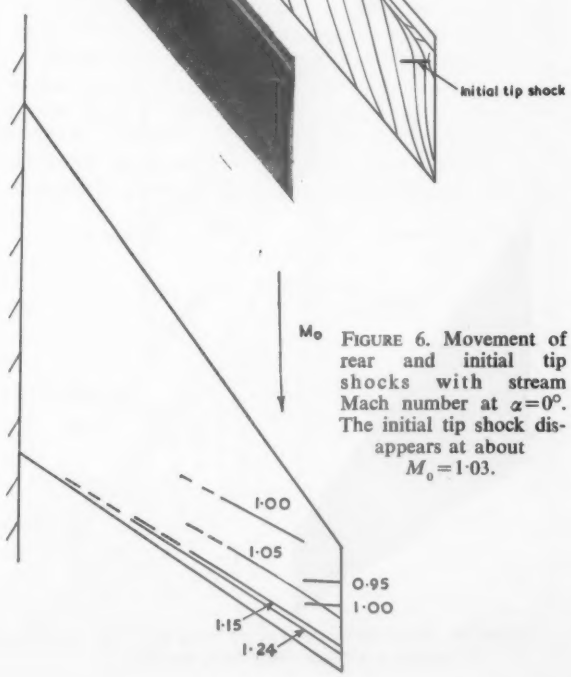


FIGURE 6. Movement of rear and initial tip shocks with stream Mach number at $\alpha=0°$. The initial tip shock disappears at about $M_0=1{\cdot}03$.

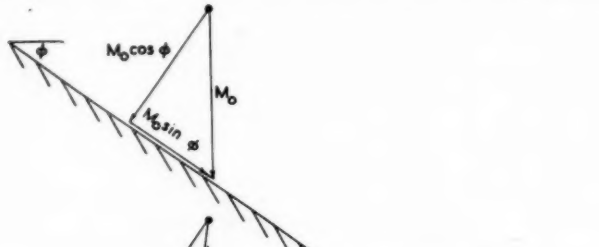


FIGURE 7. Inboard deflection of flow on surface of a swept-back wing due to local increase in velocity.

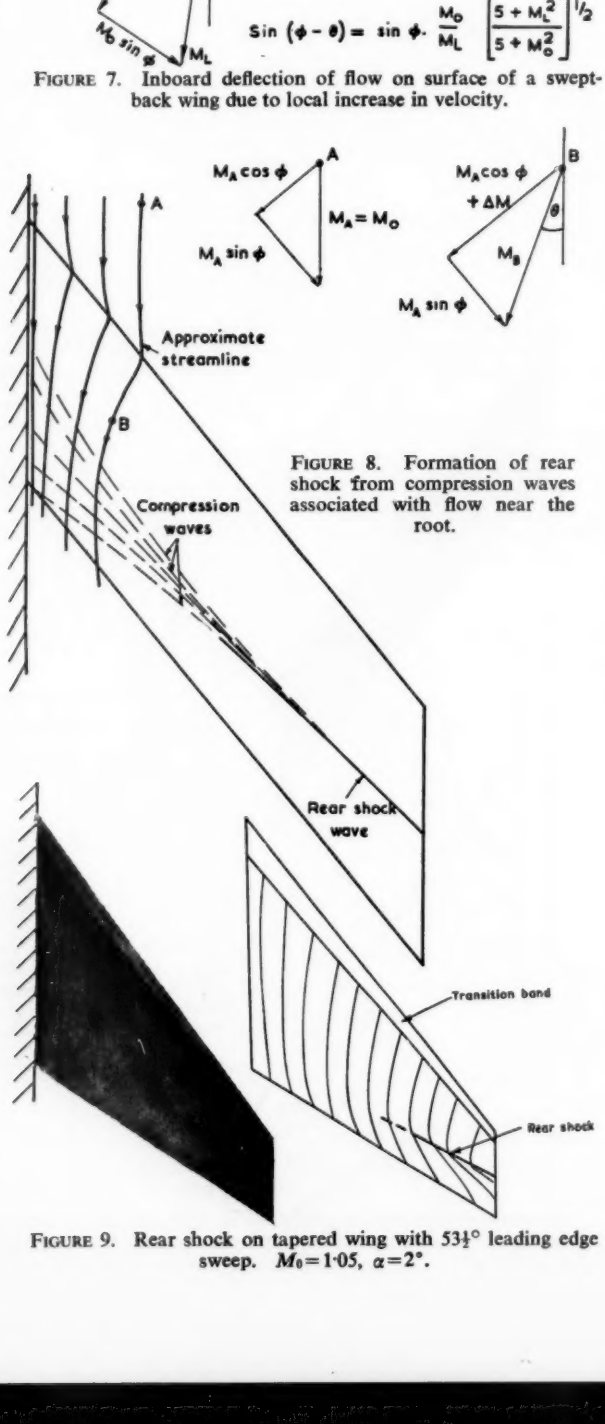


FIGURE 8. Formation of rear shock from compression waves associated with flow near the root.

FIGURE 9. Rear shock on tapered wing with 53½° leading edge sweep. $M_0 = 1{\cdot}05$, $\alpha = 2°$.

Over the inboard part of the wing, the recompression is diffuse. Fig. 10 shows the chordwise pressure distributions at four spanwise stations on the wing illustrated in Fig. 9.

In the early stages, the rear shock may propagate into the supersonic flow ahead of the initial tip shock and since, with increase in stream Mach number, the former moves rearward more rapidly than the latter (*see* Fig. 6), the initial tip shock is overtaken and disappears. This aft movement of the rear shock is accompanied by an increase in the inboard extent of the rear shock, and an associated steepening of the chordwise pressure gradient near the trailing edge farther inboard still. When the stream Mach number is increased sufficiently, the rear shock reaches the wing trailing edge and serves to return the local flow to near the free-stream pressure and direction. This condition will be reached at a lower stream Mach number in the case of a delta wing than, say, for an untapered wing with the same leading edge sweep. Because of the way the rear shock forms, its geometric sweep, with respect to the stream direction, is initially less than that of the leading edge. Its sweepback will change with Mach number and finally of course becomes that of the wing trailing edge. Because of the inward inclination of air flow mentioned earlier, the effective sweep of the shock may be somewhat lower than the geometric value.

The rear shock is a well-marked feature of transonic wing flow, contributing significantly to the wave drag. At zero incidence it is present on both wing surfaces, and may persist on the lower surface for a limited range of positive incidence. On the upper surface, it may ultimately be interfered with by the onset of leading edge separation (*see* below) but up to that stage its position on the wing surface is largely independent of incidence and is determined mainly by the stream Mach number. Moreover tests in which progressive changes were made in the leading edge shape of a 50° swept, untapered wing (ranging from a long drooped extension to a very blunt nose) showed that the leading edge geometry only had a very small effect on the rear shock position.

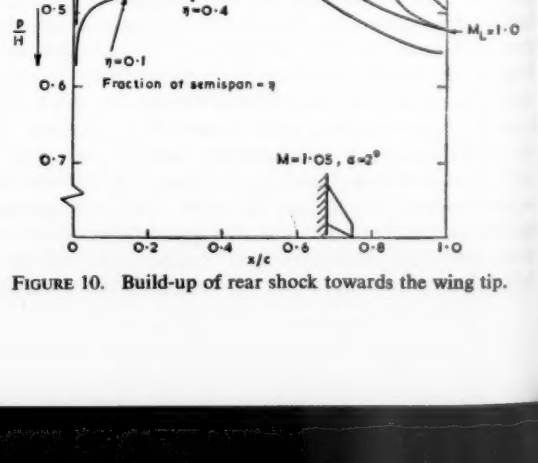


FIGURE 10. Build-up of rear shock towards the wing tip.

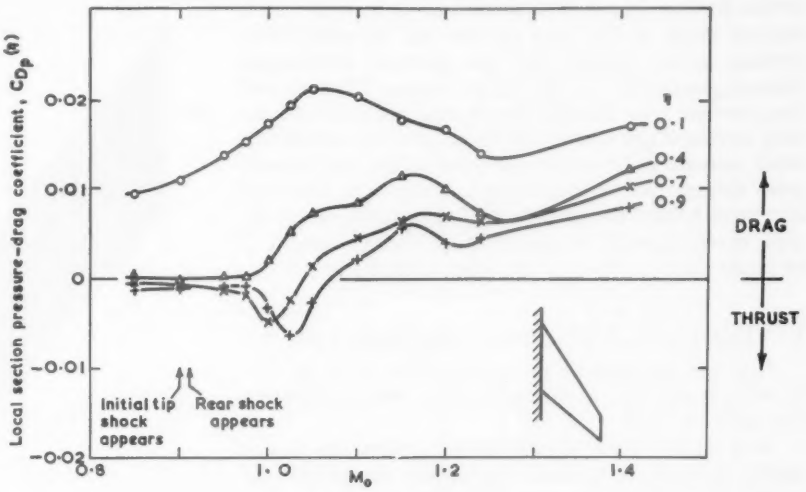


FIGURE 11. Spanwise distribution of pressure drag at zero lift.

Since the rear shock is felt to be mainly associated with root conditions, its position and strength should be strongly influenced by changes in the root contour. Those derived from area rule considerations, or by some streamline technique, tend to produce root shapes which delay the coalescence of the root compression into a rear shock, with a consequent reduction in wing drag at transonic speeds. For a given root shape this beneficial effect may be reduced as the stream Mach number rises above the design value; even at the latter condition, the influence of the root treatment may diminish towards the wing tip, and other methods may then be required to control the local flow conditions.

The local Mach number just ahead of the rear shock usually increases with the stream Mach number so that the transonic pressure freeze, so prominent in two-dimensional aerofoil flows, is not encountered*. Ultimately the rear shock may become sufficiently strong to cause separation of the boundary layer. If the shock is of low sweep, the separation will form a sheet enclosing a bubble of reversed flow. For a moderately swept shock however, the separated flow rolls into a vortex whose centre may be at a considerable angle to the stream direction. If the shock is well ahead of the trailing edge when separation occurs an appreciable part of the rear of the wing will be influenced by the vortices; as a result the surface pressure behind the shock may show a considerable pressure recovery towards the trailing edge, unlike the case of the bubble separation. Further consideration of the conditions leading to shock-induced separation will be given later.

Before turning to some of the flow phenomena associated with wings at moderate and high lift, it is perhaps worth while considering the relationship between the appearance of shock waves and the increase in wave drag. By integrating the chordwise pressure distributions obtained at zero incidence the variation with Mach number of the pressure drag at various spanwise stations may be deduced. Results for a swept tapered wing are shown in Fig. 11. At 0·7 and 0·9 of the semi-span, the section drag is negative at subsonic stream speeds and this thrust increases after the initial tip shock and rear shock have appeared, presumably because these shocks are situated ahead of the local maximum thickness position. Eventually, of course, the shocks move rearward, coalesce, increase in strength and the outer part of the wing experiences a drag. The largest contribution to the overall drag comes from the most inboard station, where the maximum local velocity is behind the crest at subsonic speeds and moves farther rearward as the Mach number becomes supersonic.

The highest value of the pressure drag coefficient attained at low supersonic speeds corresponds roughly to the condition when the rear shock reaches the wing trailing edge, and this occurs earliest at the inboard station. The subsequent rise in C_{Dp} at moderate supersonic speeds is associated with the establishment of chordwise pressure distributions which become progressively more supersonic in character.

It is of course possible to integrate the spanwise distribution of pressure drag to obtain the overall wing pressure drag, but because precise knowledge of the way C_{Dp} varies close to the root and tip is frequently lacking, the final result is not always very accurate. The overall pressure-drag coefficient obtained from the curves given in Fig. 11 is shown in Fig. 12 together with the wing drag coefficient measured on a strain gauge balance. The comparison between these two curves suggests that the

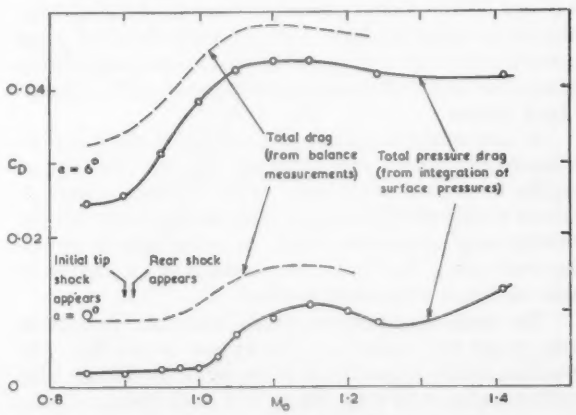


FIGURE 12. Comparison of drag curves.

*More precisely, an analogous effect has not been detected in the experiments. It has been argued that for three-dimensional flows a pressure freeze should occur very close to sonic stream speed, and is thus much more limited in extent than in the two-dimensional case. This small effect may often be masked by wall interference however and thus cannot be detected unless the model is very small compared with the tunnel. Another form of pressure freeze for the region ahead of the rear shock is discussed in Section 4·2.

surface-friction drag coefficient changes little in the Mach number range of the tests so that the transonic drag increase arises mainly from the pressure distribution changes associated with the shock waves. The overall drag increase after the appearance of shock waves on the wing surface is not at first very large, and the subsequent rapid increase occurs when the rear shock has become quite strong. At zero incidence, separation does not take place behind the rear shock at any station along the span, except possibly at stream Mach numbers near 1·2 when the shock is close to the wing trailing edge.

3. The Flow at Moderate and High Lift

It is now appropriate to consider how the flow over the wing is modified as the incidence is increased from a low value.

At a stream Mach number of 0·7, where the flow is wholly subsonic at a small incidence, an increase in incidence causes the local flow velocities near the leading edge to rise along the entire span. The most rapid changes occur near the tip however, due to the forward shift in loading on that region, and a somewhat higher lift-curve slope. The intense pressure gradients which must therefore exist in the leading edge region lead ultimately to flow separation, which usually starts at the tip and moves progressively inboard with increasing incidence.

With leading edge sweep angles greater than about 40° the separation rolls up to form a part-span vortex lying obliquely across the wing. The typical helical form of the surface flow beneath the vortex and the characteristic reattachment line are shown in Fig. 13. The position of the vortex on a wing is largely independent of Mach number, provided this is below a certain critical value to be discussed later, and depends mainly on the wing incidence. As this is increased the vortex moves inboard and increases its sweep (Fig. 14).

The incidence at which leading edge separation first occurs is dependent among other things on the leading edge shape of the wing section. In Fig. 15, the separation boundaries found for four different modifications of a basic 50° swept-back wing are shown. At present we are concerned with the horizontal or slightly sloping parts of the boundaries for stream Mach numbers between 0·6 and 0·95. These correspond to the leading edge separation extending to 0·1 of the local chord at some spanwise position. It is apparent that leading edge droop delays the onset of separation compared with a sharp-edged section.

At low subsonic speeds an appreciable delay can be achieved by using a very blunt section, but this type of profile is subject to a large Mach number effect. It should also be mentioned that with a sharp-edge section, leading edge separation tends to occur almost simultaneously along the whole span, and not just at the tip, as with moderately-rounded profiles.

The precise conditions which stimulate separation from swept-back edges are not known at present. The problem clearly resembles in many ways the leading edge stall on an unswept wing, which is in itself quite complex. It can be shown however that the progressive development

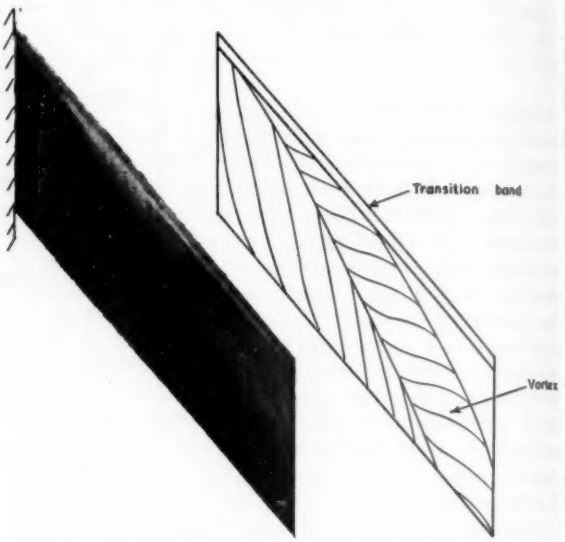


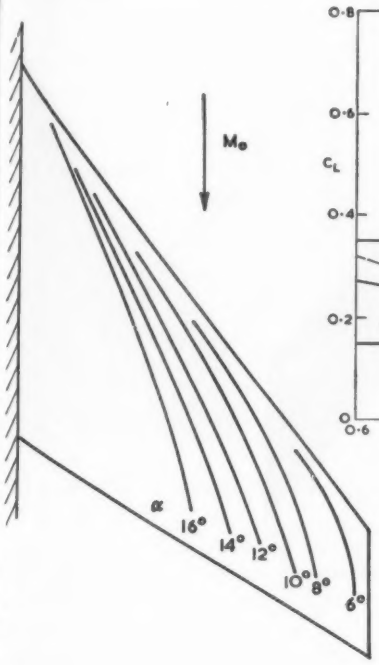
FIGURE 13. Part-span vortex on 50° swept-back wing; $M_0 = 0{\cdot}80$, $\alpha = 12°$.

of the separation along the span of a swept-back wing with a moderate leading edge radius is associated with the attainment of certain flow conditions at each station. For example, in Fig. 16 separation occurs close to the tip at about 5° incidence. At incidences above this, the lowest pressure in the first one per cent of the chord (and hence over the entire wing) occurs at spanwise positions which are progressively nearer the root, and the value of this minimum pressure is almost independent of incidence. The peaks in the curves of Fig. 16 can in fact be related to the inboard spread of leading edge separation and hence the effective origin of the vortex.

Those parts of the wing lying immediately beneath the centre of the vortex experience a local reduction in pressure due to the high local air velocities. This effect diminishes or may even reverse as the reattachment line is approached. Thus a particular spanwise station undergoes an increase in lift-curve slope as the vortex moves inboard with rising incidence; this changes to a rapid fall as the secondary separation line (the outboard edge of the vortex) moves inboard of the station which is then left within a dead-air region of almost constant pressure.

The sequence is illustrated for the two outboard stations of the swept tapered wing of Fig. 17. At 0·4 semi-span, the extra lift due to the vortex is present at the upper end of the incidence range, which is not however large enough to show the subsequent rapid fall.

As the stream Mach number is raised, the rear shock will appear and, as was mentioned earlier, its position on the wing surface is largely independent of incidence. It may be present simultaneously with the leading edge separation and its attendant vortex, and can then usually be detected between the root and the vortex reattachment line in the oil-flow pattern. Moreover there may well be a marked intersection between the rear shock and the



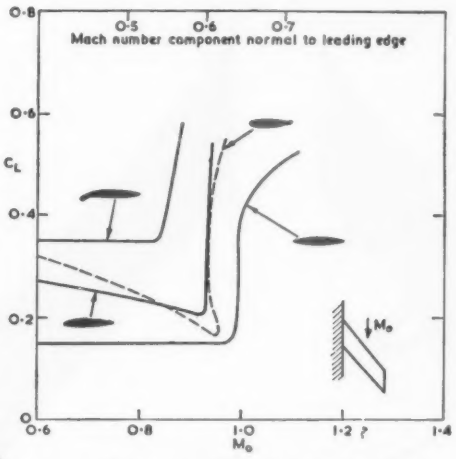


FIGURE 15. Separation boundaries for 50° untapered wings with various leading edge geometries.

FIGURE 14 (*left*). Approximate locations of the vortex centre on the upper surface of a swept wing at $M_0 = 0{\cdot}6$. Moderate leading edge radius.

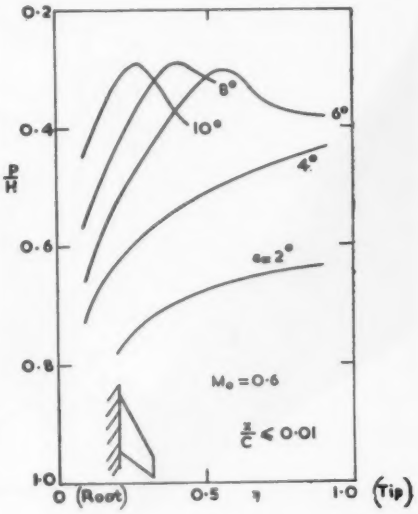


FIGURE 16. Spanwise variation of peak suction in leading edge region.

vortex, perhaps similar to that shown in Fig. 18. The shock is here considered to extend through the vortex and to cause an extension of the secondary separation line outboard, as well as a widening of the vortex to the rear of the shock.

With further increase in stream Mach number a change occurs in the flow pattern. Separation no longer takes place at the leading edge; the flow remains attached to the wing surface as far back as a shock wave, which appears to originate close to the leading edge at some spanwise position and to turn back over the wing surface outboard of this station. Flow separation may still take place at the shock, but the region of attached flow between the leading edge and the shock increases with both stream Mach number and incidence, as the shock moves rearward and inboard.

This transition from separated to attached leading edge flow has been called the transonic flow attachment and its occurrence is represented by the almost vertical boundaries of Fig. 15. Over a wide range of incidence there is little change in the stream Mach number at which the attachment takes place, though as can be seen leading edge shape exerts a considerable influence.

At Mach numbers and incidences above those shown in Fig. 15, the boundaries become less steep, but still continue to increase steadily with rising Mach number. In this region, they correspond to the beginning of a tip separation which spreads inward with increasing incidence, until ultimately the flow separates along the entire leading edge. A part-span vortex, very similar to that present at the same incidence at subsonic speeds, then dominates the wing flow.

The component of stream Mach numbers normal to the wing leading edge for the flow attachment condition varies between about 0·55 and 0·90, depending on the

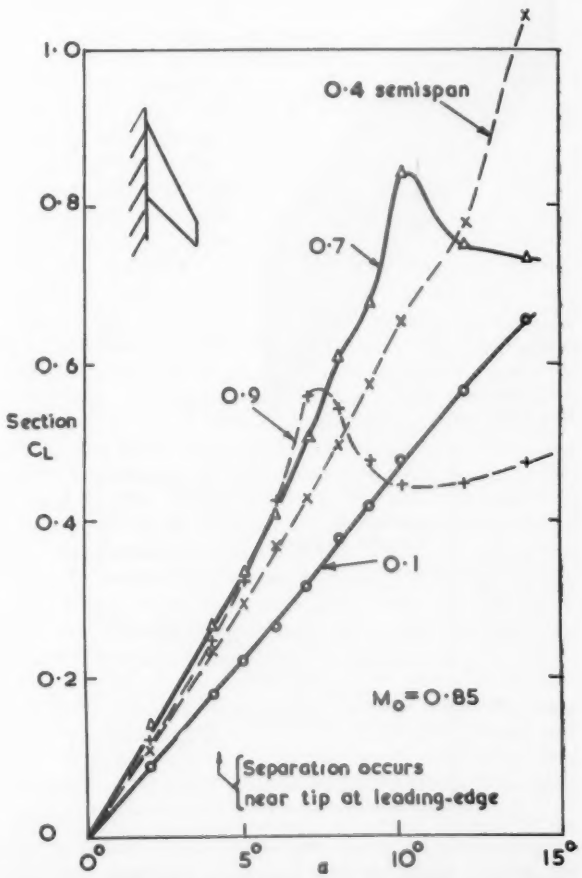


FIGURE 17. Local section lift curves.

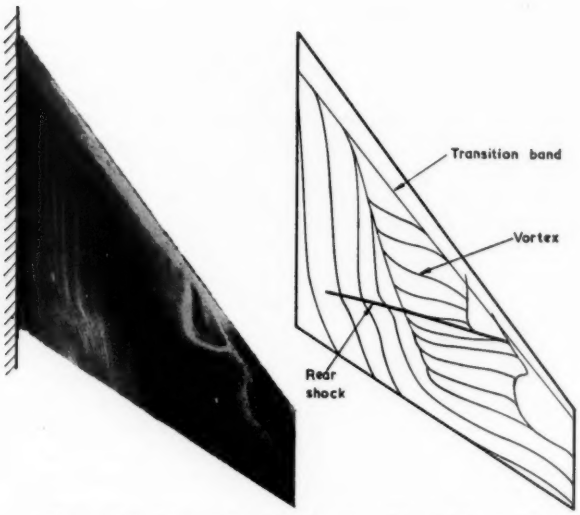


FIGURE 18. Vortex shock interaction, $M_0=0{\cdot}95$, $\alpha=11°$.

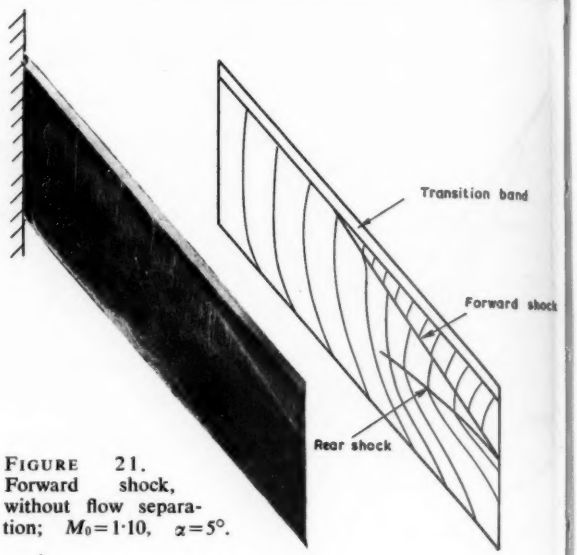


FIGURE 21. Forward shock, without flow separation; $M_0=1{\cdot}10$, $\alpha=5°$.

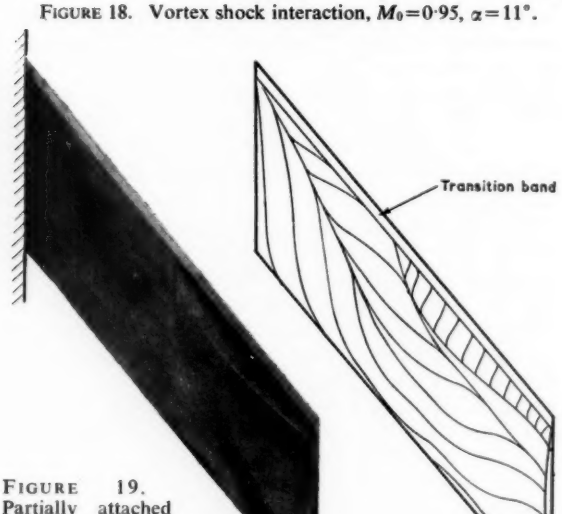


FIGURE 19. Partially attached leading edge flow on 50° swept wing with sharp leading edge. $M_0=1{\cdot}10$, $\alpha=9°$.

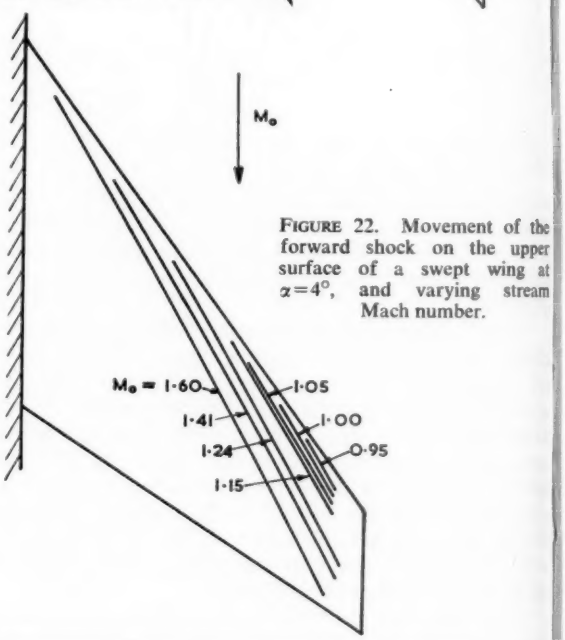


FIGURE 22. Movement of the forward shock on the upper surface of a swept wing at $\alpha=4°$, and varying stream Mach number.

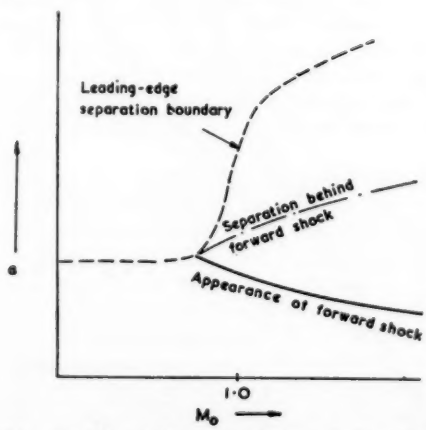


FIGURE 20. Sketch of flow boundaries associated with the forward shock.

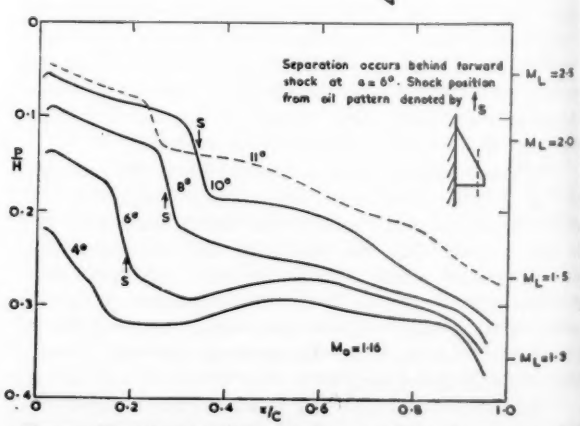


FIGURE 23. Upper-surface pressure at 0·7 semi-span, showing forward shock movement.

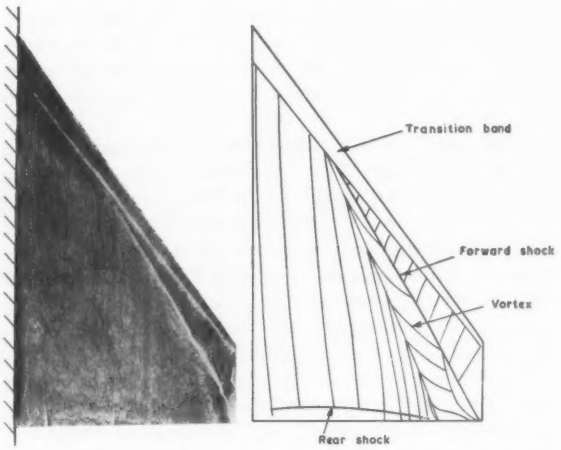


FIGURE 24. Forward shock with separation; $M_0=1{\cdot}16$, $\alpha=8°$.

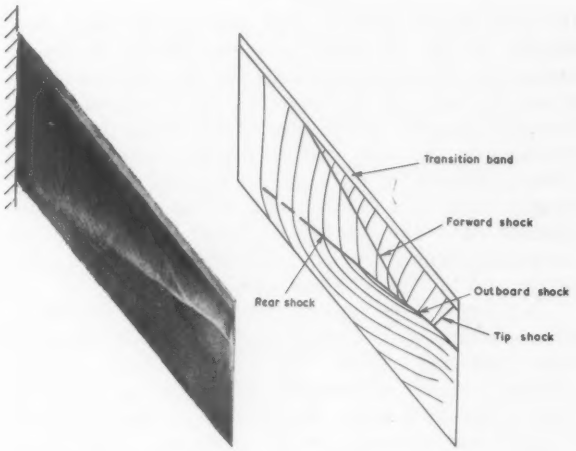


FIGURE 25. Appearance of outboard shock; $M_0=1{\cdot}10$, $\alpha=9°$.

wing sweep and particularly the leading edge shape. Attempts to relate observed values with the corresponding flow change over on a two-dimensional aerofoil having the same section as that normal to the wing leading edge have not proved very successful, except for certain sharp-edged profiles.

Swept wings with sharp leading edges may exhibit a type of flow in which leading edge separation (forming a part-span vortex) exists over the inner part of the wing and attached leading edge flow back to a shock wave (which itself causes separation) occurs over the outer part. An example is shown in Fig. 19. This flow arrangement is of course the opposite to that found on wings with moderately-rounded leading edges, where leading edge separation takes place over the outer part of the span. As the incidence of the sharp-edged wing is increased, the region of attached flow contracts and ultimately disappears, so that separation exists along the entire leading edge. On a wind tunnel model (and perhaps in flight) this final disappearance of the attached flow may cause a violent oscillation of the wing, akin to stalling flutter, because of an interaction between the twisting of the model due to the aerodynamic forces and the large loading changes which take place at the tip during the transition from attached to separated flow.

The shock wave which has been mentioned in connection with leading edge attachment has been called the forward shock. It is not, however, necessarily connected only with the attachment process, but arises naturally from the local conditions near the leading edge and can thus appear at incidences well below that at which leading edge separation takes place. The boundary along which the forward shock first appears for a wing of about 50° sweep is sketched in Fig. 20. The position of this boundary is greatly affected by the leading edge shape, occurring at a higher lift if the leading edge is drooped. The forward shock will also appear at a lower incidence on a very blunt section than on one having a sharp leading edge. A typical oil pattern is shown in Fig. 21. The forward shock movement with stream Mach number at a fixed incidence is illustrated by Fig. 22; the shock wave remains comparatively straight with little change in the mean geometric sweep. The movement of the shock is due mainly to an inboard shift of the position at the leading edge from which the shock turns back over the wing. It must not be supposed that the shock does not exist at the leading edge inboard of this position, though it may well fade away before the wing root is reached.

A similar shock movement to that shown in Fig. 22 occurs if the wing incidence is increased at constant stream Mach number. At a given spanwise station, the forward shock would appear to move towards the trailing edge, as indicated by Fig. 23.

The pressure rise associated with the shock is quite large at transonic speeds and the overall static pressure ratio across the shock increases with incidence, as does the local Mach number just upstream of the shock. At some stage the forward shock becomes strong enough to cause separation of the boundary layer, which rolls into a vortex behind the shock (Figs. 20 and 24). This has the effect of reducing the pressure ratio across the shock, partly because of the low pressures induced on the wing surface beneath the vortex. The pressure distribution at $\alpha=11°$ in Fig. 23 corresponds to the development of a large vortex behind the forward shock, whose effect extends over all the pressure plotting station except ahead of the forward shock. The flow is about to separate along the entire leading edge, and as a result a forward movement of the shock has started to take place.

It is possible for the rear shock and the forward shock to intersect on the wing surface as in Fig. 25; the shock between this intersection and the wing tip has been called the outboard shock*. The appearance of the outboard shock on a wing of about 50° sweep is of considerable importance, because it always seems to cause flow separation, even if this is absent for the forward and rear shocks separately. This fact is not surprising since the pressure rise through the outboard shock is roughly

*In some cases the surface oil patterns indicate the trace of the forward shock as it passes beyond the intersection point behind the outboard shock (*see* Fig. 28). The forward shock is above the separated fluid however and does not appear to influence the surface pressures.

the sum of the pressure rises through the other two shocks, neither of which may be sufficient to cause separation. The geometric sweep of the outboard shock is usually small and this angle is further reduced by the inward inclination of the flow ahead of the shock. Its position on the wing surface depends largely on the relative position of the rear and forward shock and a typical variation is shown in Fig. 26 for a stream Mach number of 1·41. As the incidence is increased the separation behind the outboard shock becomes more severe and the shock moves forward.

It is not without interest to note that this separation occurs in the region in which there may be an aileron or other part-span control. The initial appearance of the outboard shock is strongly influenced by the wing section and plan form. For example, if the wing span is small the appearance of the outboard shock will be delayed until a higher stream Mach number, compared with a large-span plan form. Moreover if the wing trailing edge is unswept the rear shock will reach this earlier than if it is swept and the intersection between the rear and forward shocks will then probably take place at the trailing edge. Even if the subsequent shock movement with incidence is similar to that illustrated in Fig. 26, the area of separated flow may well be less than if the intersection takes place closer to the leading edge.

Mention has been made so far of four shock waves which may appear on the wing surface; the initial tip shock, and the rear, forward and outboard shocks. To these must be added the tip shock, which is a disturbance within the local supersonic flow, associated with and originating close to the tip leading edge and which indicates the inboard limit of the wing surface influenced by the tip. At low incidences this shock is comparatively weak, but an increase in strength occurs as the lift increases. This effect may be associated with the increasing strength of the tip vortex as well as the increasing difference in flow conditions between the wing upper surface at the tip and the air just outboard.

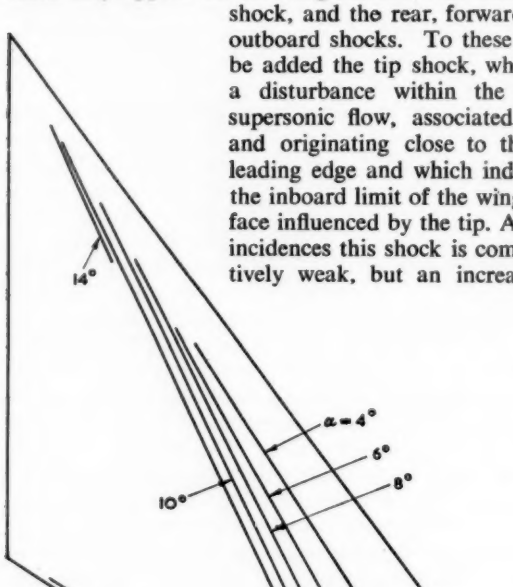


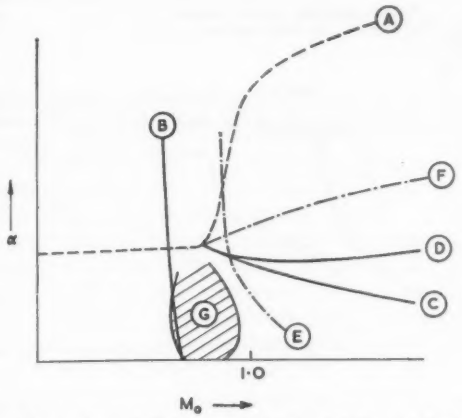
FIGURE 26. Outboard shock movement on upper surface at $M_0 = 1{\cdot}41$.

The onset of tip separation at supersonic stream speeds and high wing incidences often appears to correspond to a separation of the flow as it passes through the tip shock. As the separation moves inboard along the wing leading edge, it carries the tip shock with it to form the demarcation between the attached and separated regions. On a cropped delta plan form tested at the N.P.L., flow separation took place in the region behind the intersection of the tip and forward shocks though in this case the flow was complicated by the presence of the rear shock at the wing trailing edge, which was not far behind the intersection. For the class of wings considered in the present paper, the tip shock does not show up clearly on surface oil-flow patterns, except when tip separation is present. If the leading edge sweep is reduced however, the shock becomes more prominent. In the case of a finite unswept wing, the forward and tip shocks (indicating the wing regions subject to root and tip effects) become very similar.

It should not be forgotten that the bow shock wave forms ahead of the wing as soon as the stream Mach number exceeds unity. At low and moderate transonic speeds this exerts only a small influence on the flow over the wing surface and it is convenient to neglect the changes in flow direction and stream total pressure that it causes.

The main features of the wing flow have now been presented and can be briefly summarised. The effect of sweep on the distribution of pressure over the wing surface, combined with the constraint imposed by the wing root, lead to the formation of a rear shock lying obliquely across the wing. Conditions close to the tip produce a comparatively weak initial tip shock, whose subsequent development is prevented by the aft movement of the rear shock. As the incidence is increased at constant Mach number, high velocities occur locally near the leading edge and at subsonic stream speeds flow separation ultimately develops leading to the formation of a part-span vortex. With increase in stream Mach number this separation is suppressed and a forward shock appears, running obliquely across the wing from some position along the leading edge. The appearance of the forward shock depends on local conditions at the leading edge (a matter discussed in more detail later) and is not primarily associated with the transonic flow attachment. The forward shock moves inboard and rearward with increasing wing incidence and stream Mach number and ultimately may intersect the rear shock, whose position across the wing is largely dependent on the stream Mach number alone. The outboard shock exists on the tip side of the intersection and is usually strong enough to provoke flow separation. This may also occur at some stage behind both the rear and forward shocks separately.

At a high enough incidence, leading edge separation takes place close to the wing tip, possibly due to the increasing strength of the tip shock and spreads inboard



(A) Leading edge separation.
(B) Appearance of rear shock.
(C) Appearance of forward shock.
(D) Appearance of outboard shock, with separation to rear.
(E) Separation behind rear shock.
(F) Separation behind forward shock.
(G) Zone in which initial tip shock exists.

FIGURE 27. Typical boundary diagram for wing of about 50° sweep.

until flow separation occurs along the entire leading edge. The flow pattern on the wing surface is then dominated by a large part-span vortex and closely resembles that for the wing at subsonic speeds and high incidence.

The various flow boundaries appropriate to a wing of about 50° sweep are sketched in Fig. 27. In Fig. 28, an oil-film pattern is presented which was obtained on an untapered wing at high incidence and a slightly subsonic stream Mach number, and which illustrates many of the features of transonic wing flow. The forward and outboard shocks are clearly visible, but the rear shock is more diffuse, particularly near the wing root. Nevertheless, this shock causes a marked spanwise drift of the boundary layer to the rear. The flow approaching both the outboard and forward shocks is attached and inclined towards the root. The boundary layer separates at these shocks, a small vortex being formed behind the forward shock and a large well-defined vortex existing to the rear of the outboard shock. The latter vortex is also fed from the leading edge separation close to the tip.

This figure illustrates the complexity which is possible in transonic flows about swept-back wings and the departure of the real flow from the model assumed in simple sweepback theory.

4. Some Flow Problems arising with Swept-back Wings of Transonic Speeds

Having established the main features of the flow development about moderately swept-back wings, it is perhaps pertinent to consider a few aspects of the surface flow in more detail. Specifically we shall consider briefly the origin of the forward shock, the nature of the flow fields in the regions ahead of and behind the forward shock, and finally the conditions that seem to be required to promote shock-induced boundary layer separation.

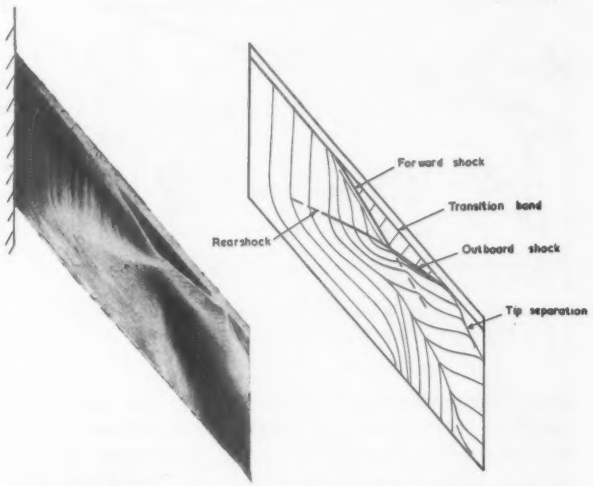


FIGURE 28. Complex flow pattern on 50° swept wing; $M_0 = 0{\cdot}95$, $\alpha = 10°$.

4.1. THE ORIGIN OF THE FORWARD SHOCK

For wings having a moderate leading edge radius the chordwise and spanwise gradients in the region ahead of the forward and outboard shocks are comparatively small. This is fortunate because the direction of an oil-film filament on the wing surface in this region can then be assumed to be the air flow direction just outside the boundary layer. An approximate check can be made on this assumption by comparing the observed values of the flow deflection θ, with those calculated from the local Mach number, using the simple flow-model sketched in Fig. 7. Typical results are shown in Fig. 29. The correlation is satisfactory at transonic speeds in view of the assumptions involved and the difficulties in measuring the filament directions on the oil-flow photographs.

Knowledge of the flow inclination ahead of the forward shock enables a tentative view of the origin of this shock to be tested. If it is accepted that the forward shock derives from the pressure disturbance associated with the leading edge, the shock should propagate at

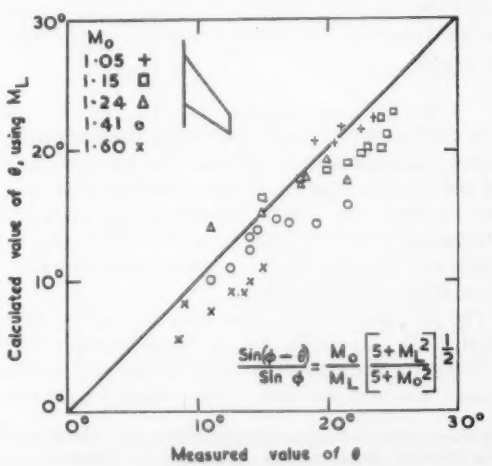


FIGURE 29. Flow inclination ahead of the forward shock for a tapered wing with 53¼° leading edge sweep.

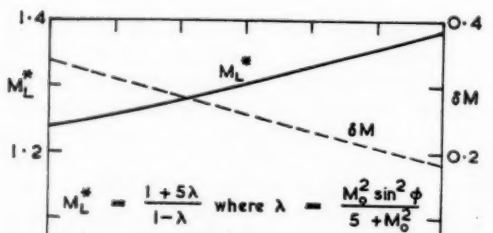


FIGURE 30. Critical conditions for appearance of forward shock on wing of 50° leading edge sweep.

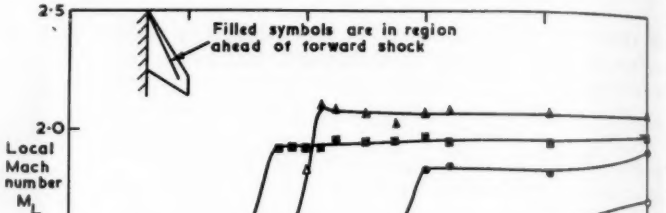


FIGURE 32. Local Mach number variation at a given chordwise station.

nearly the appropriate Mach angle to the local flow direction. If the outward running Mach line lies behind the wing leading edge, the shock will appear on the surface. This is merely equivalent to a condition that for a given leading edge sweepback and stream Mach number, the local Mach number in the leading edge region must be greater than a certain critical value, M_L^*. Because of the inward deflection of the flow M_L^* is much lower than would be the case if the flow over the wing surface remained in the stream direction. A simple and approximate relation for M_L^* and its variation with stream Mach number is given in Fig. 30.

This critical Mach number will appear first at the wing tip and be attained at progressively further inboard stations as the incidence is increased at constant stream Mach number. The effective origin of the forward shock (or point at which it appears to move back from the leading edge) should move inboard too, as in fact indicated in Fig. 22. From a knowledge of the way the peak local Mach number close to the leading edge varies with incidence and stream Mach number, it is possible to predict the wing incidence at which the forward shock should first become visible at a given spanwise station, using the simple flow model described. This can be compared, as for example in Fig. 31, with an estimate of the forward shock appearance based on sequences of oil-flow patterns. The agreement shown in Fig. 31 is remarkably good, despite the obvious approximations that are involved, and gives confidence to the proposed mechanism.

4.2. TYPES OF FLOW FIELDS

It follows from Section 4.1 that the forward shock divides two types of flow region. That between the shock and the leading edge is virtually isolated from the wing root. In this, the region resembles that ahead of the disturbance from the apex of a flat delta wing with a supersonic leading edge*, a zone with marked two-dimensional characteristics.

The pressure at some point near the leading edge falls rapidly with increasing stream Mach number (Fig. 32) until the forward shock moves to the rear. The subsequent change in pressure (or local Mach number) is then quite small. This behaviour is obviously similar to that exhibited by a point on the surface of a two-dimensional aerofoil at transonic speeds, where a "pressure freeze" occurs once the point lies within the supersonic region ahead of the shock.

While the region ahead of the forward and outboard shocks can therefore be regarded as behaving similarly to what would be expected from two-dimensional results, the region behind the forward shock is markedly different. The local Mach numbers continue to rise with increasing stream Mach number and as the rear shock moves aft; no transonic pressure freeze occurs. This region however appears to have the property that the ratio of the local pressure to the free-stream static pressure remains constant at transonic speeds, the upper-surface pressure distribution collapsing as shown in Fig. 33. This is a much better correlation than would be obtained by use of the conventional pressure coefficient C_p.

At low and moderate supersonic speeds, the ratio of the pressure at the surface of a slender cone to the free-stream static pressure remains constant as the stream Mach number changes, and this suggests that the flow over the wing in the region between the forward and rear shocks, and the root, is partially conical in type. Support for this view can be obtained from the pattern made by the velocity contours in this region, which lie close to rays passing through the root leading edge; there is a marked similarity between this velocity field and that

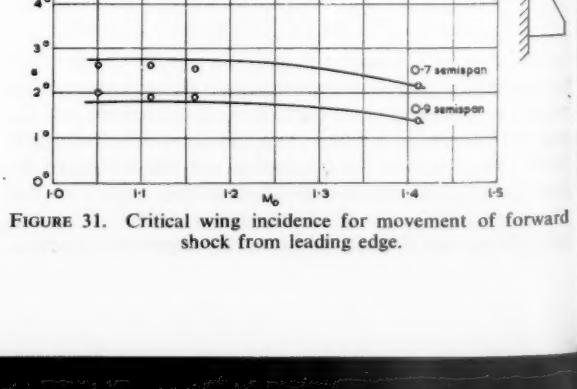


FIGURE 31. Critical wing incidence for movement of forward shock from leading edge.

*The resemblance should not be pressed too far. The region ahead of a forward shock may be influenced by disturbances which propagate along the lower surface from the root, say, and pass around the leading edge onto the upper surface.

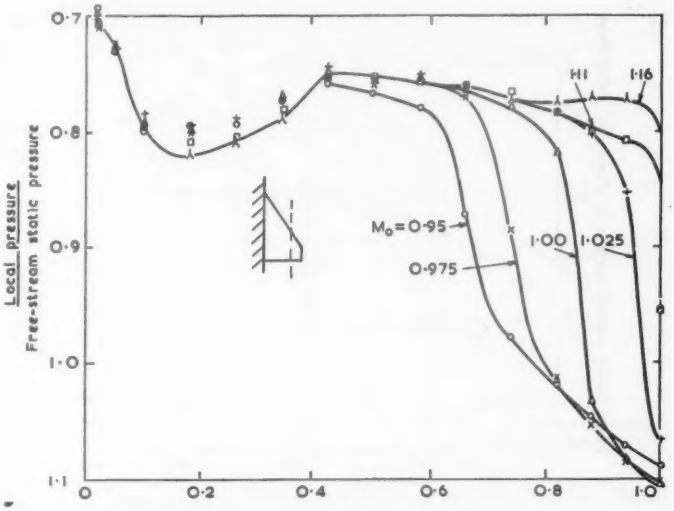


FIGURE 33. Upper surface pressure distributions at $\alpha=4°$, $\eta=0{\cdot}70$.

between the surface of a cone and the shock wave from the vertex. In this analogy the wall (or plane of symmetry) at the wing root is equivalent to the cone surface and the vertex shock is represented by the forward shock.

It has already been remarked that the pressure gradients are very small in the region ahead of the forward shock wave. Therefore the flow there may also be considered to be approximately conical if desired. Considering the two regions together it can be seen that the forward shock wave may be regarded as being similar to the disturbance from the apex of a flat wing with a swept, supersonic leading edge.

4.3. BOUNDARY-LAYER AND SHOCK-WAVE INTERACTION ON SWEPT WINGS

In recent years great attention has been paid to the problems arising from the interaction of shock waves and boundary layers, including a study of the conditions which promote boundary-layer separation. Most of this effort has been concerned with two-dimensional flows on aerofoils or flat plates. The shock waves which occur on swept-back wings may be used in order to study cases where the flow approaches the shock wave obliquely. In the present work, the surface oil-film has been used to indicate the local flow direction as it approaches the forward shock wave and also the onset of flow separation to the rear of the shock. The latter event is assumed to take place when the oil filament immediately behind the shock wave lies parallel to it; in other words the flow separation and subsequent reattachment are almost coincident.

An analysis of the results so far obtained on a limited number of wings with leading edge sweepback of about 50° suggests that the separation of a turbulent boundary layer through the forward shock occurs when the component of local Mach number normal to this shock exceeds about 1·39. This criterion, which must be regarded as tentative until further confirmation is obtained, seems to be independent of wing incidence, stream Mach number, local Mach number just ahead of the shock and shock position. Results for two wings with the same leading edge sweep are shown in Fig. 34. The pressure ratio across the forward shock at separation varies with both stream Mach number and spanwise position as the inset to Fig. 34 shows.

This critical Mach number component is rather higher than the corresponding value of about 1·25 deduced by Pearcey and others at the N.P.L. from tests on two-dimensional aerofoils with turbulent boundary layers upstream of the shock. It seems reasonable to attribute the difference in the two values to the three-dimensional nature of the wing flow including the formation of a vortex from the separated flow; a significant effect of shock sweep on the separation condition therefore seems to exist, and indeed this problem is at present being investigated in a more simple and specially-designed experiment.

It might be argued, however, that the outboard and rear shocks on the swept wing provide material to study

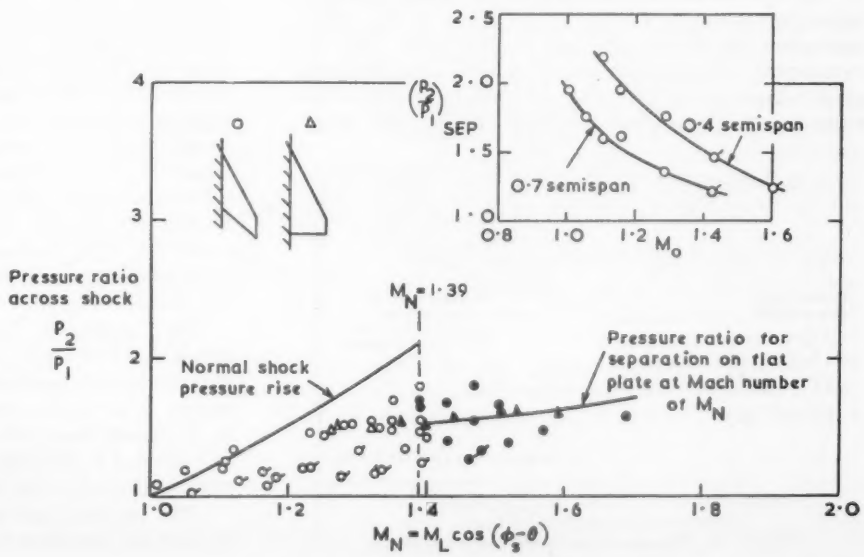


FIGURE 34. Separation at forward shock for two wings with 53½° leading edge sweep. Presence of separation is denoted by filled symbol. Plain symbols denote stream Mach number range between 1·0 and 1·24; symbols with tails are for Mach numbers of 1·41 and 1·60.

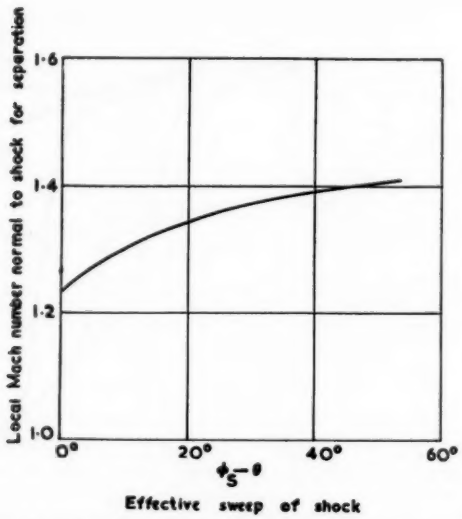


FIGURE 35. Possible effect of shock sweep on separation condition.

separation effects for shock of lower sweep. Unfortunately the direction of the oil filaments ahead of the rear shock do not give such a reliable indication of the local flow direction as those ahead of the forward shock, due to the larger transverse pressure gradients. Nevertheless, an estimate of the separation condition can be made in certain cases and for rear shocks, whose effective sweep relative to the oncoming local flow is about 20°, the critical Mach number component is about 1·34. In the case of a delta wing where the rear shock was almost completely unswept at the separation condition, this value was reduced to about 1·23, which is the lower bound of the corresponding two-dimensional results. The influence of shock sweep on the separation may therefore be similar to that indicated in Fig. 35. The effective sweep of the forward shock at separation is usually about 40° for the present type of wing.

The outboard shock provides little information on the separation conditions since in all tests so far made separation has always occurred at this shock, and the component of local Mach number normal to it has been greater than about 1·7. The pressure ratio across the outboard shock is plotted against the normal Mach

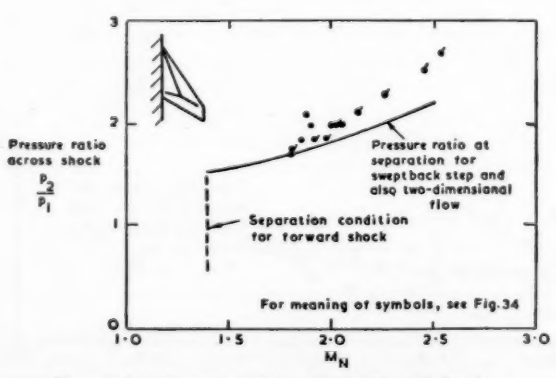


FIGURE 36. Pressure ratio across outboard shock.

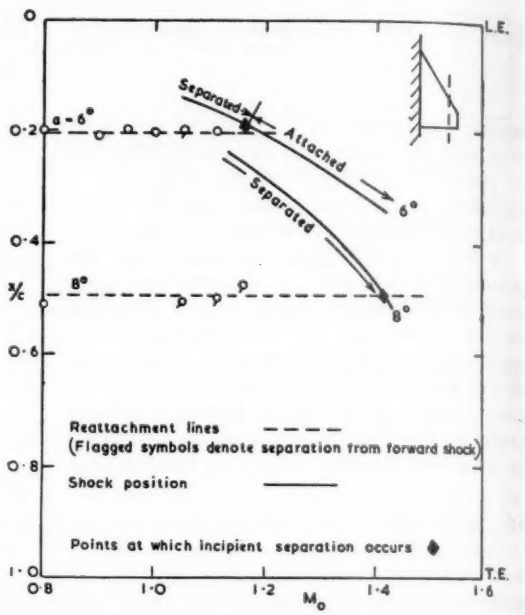


FIGURE 37. Flow attachment conditions for cropped delta half-wing; 0·7 semi-span.

number component in Fig. 36, and it is interesting to note that these results are not greatly different from those obtained from swept-back steps, and two-dimensional flows.

It would appear from the foregoing that despite inherent inaccuracies in determining the local flow direction, and deciding the exact local conditions corresponding to the beginning of shock-induced separation, considerable progress has been made in showing that the value of the local Mach number component normal to the shock determines the onset of flow separation.

It is somewhat embarrassing therefore to note that an equally successful, but apparently unrelated, criterion for shock-induced separation at the forward shock can be deduced from the experimental data. This is derived from the fact that the position of the reattachment line on the wing surface away from the immediate tip region appears to depend only on wing incidence and is largely independent of the stream Mach number or whether the separation has occurred at the wing leading edge or from a forward shock. This phenomenon is illustrated by the broken line in Fig. 37, for two typical values of the incidence. The forward shock, however, moves rearward with increasing Mach number (as indicated by the full lines).

Flow separation, which is present at the lower stream Mach numbers, ceases when the shock position coincides with the reattachment line position, and attached flow occurs at all higher Mach numbers for this incidence. It is strange that this change should also correspond to a value of 1·39 being attained for the component Mach number normal to the shock front.

The method just described may be considered as seeking the condition for flow reattachment, as opposed

to the earlier approach of studying the conditions for flow separation, and hence coincident results should be obtained at incipient separation. Nevertheless the relationship between the two approaches remains obscure and it is probable that they represent different aspects of a deeply underlying unity, as yet undiscovered.

5. Effect of Flow Development on Wing Forces and Moments

In the preceding sections, the main features of the transonic flow about wings of moderate leading edge sweep have been discussed, together with some more detailed discussion of certain flow problems. From the practical point of view the influence of the changing flow pattern of the wing lift, drag and pitching moment is of great importance and will now be considered briefly. It is perhaps fortunate that this effect is rather smaller than might be expected. For example the boundaries representing leading edge separation, and separation behind the forward shock (*see* Fig. 20) have been superimposed on a carpet of lift curves in Fig. 38. The development of the vortex flow over the wing at high incidences leads to an initial slight increase, and a subsequent decrease, in lift-curve slope, but the effect is not large. Indeed the carpet is of a most regular form, the most significant feature being the rise in lift coefficient at constant incidence as the stream Mach number is increased from 0·95 to 1·05. This corresponds to the growth and movement to the trailing edge of the rear shock, and is accompanied by a marked rearward shift in the wing centre-of-pressure position.

With this cropped delta plan form, the pitching moment curves remain linear with increasing lift (Fig. 39) despite the development and movement of the part-span vortex. If the wing trailing edge is swept however, this phenomenon may well lead to a non-linear pitching moment curve with a marked pitch-up at high lift. The latter can be particularly severe at moderate supersonic Mach numbers if the tip separation spreads inboard rapidly.

The relationship between the transonic drag rise and the appearance of the initial tip and rear shock has been discussed in Section 2 for the zero-lift cases and will not be considered further. For lifting conditions the wing drag polars are not very revealing, except for example

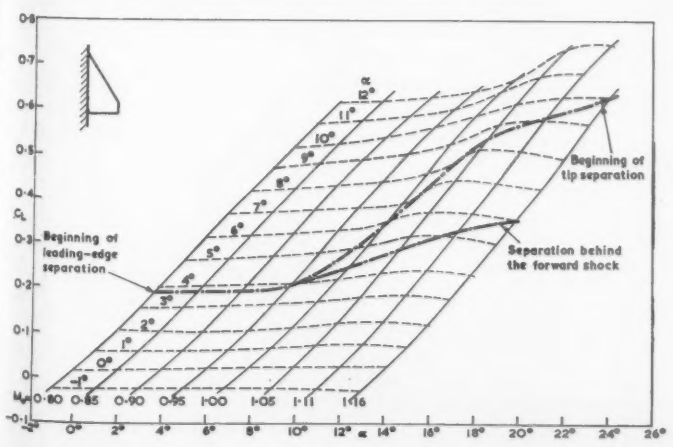


FIGURE 38. Lift results for cropped delta half-wing (from balance tests).

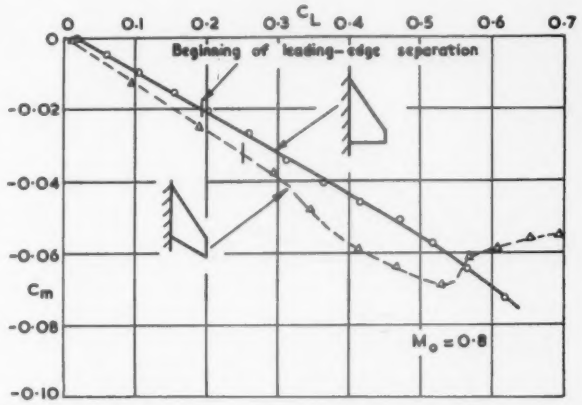
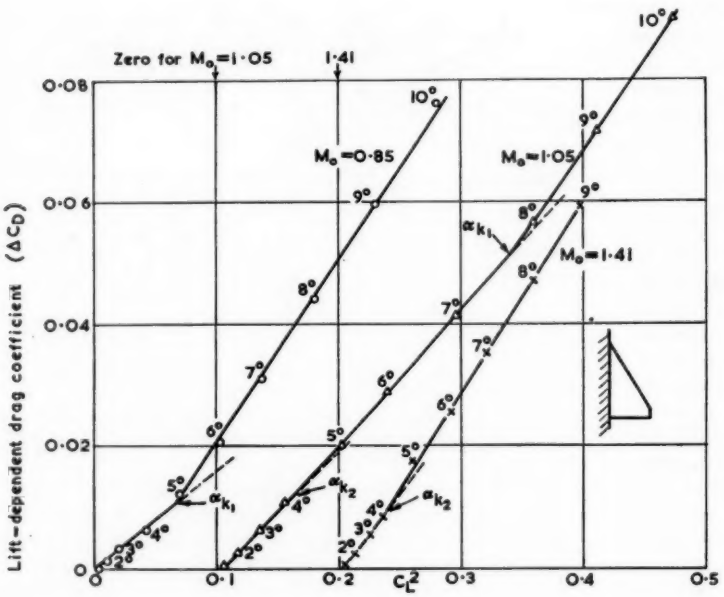


FIGURE 39. Effect of plan form and flow separation on wing pitching moment about quarter mean aerodynamic chord.

when they show that a reduction of drag and lift occurs for a sharp-edged wing when at constant incidence and increasing stream Mach number the leading edge flow becomes attached over the outer part of the span, as illustrated in Fig. 19. There are many ways of analysing lift-dependent drag however and one valuable procedure is to plot the increase in drag coefficient due to lift (ΔC_D) against C_L^2. The resulting curves can often be represented with considerable accuracy by a few straight lines, of different slope, as illustrated in Fig. 40. At subsonic stream speeds, two lines are generally required and the incidence corresponding to the intersection of these, and hence roughly representative of the position of some flow change, has been labelled αk_1. This kink incidence remains roughly constant until a rapid rise takes place as the stream speed approaches and exceeds the sonic value. In fact the change in slope of the lift-dependent drag curve associated with αk_1, corresponds to the onset of leading edge separation, as Fig. 41 shows.

At supersonic Mach numbers, three straight lines are frequently required, and a second kink incidence (αk_2) may be obtained at a lower value than αk_1. The significance of αk_2 is less easy to assess since it appears to correspond to a stage in the flow development after the appearance of the forward shock, but before this has become sufficiently strong to cause flow separation. In reality, the change in the lift-dependent drag conveniently represented by αk_2 is spread over a small range of incidence and most probably arises from a significant loss in leading edge suction due to the inward movement of the forward shock.

Since the lift-dependent drag may be appreciably greater when leading edge flow separation is present, changes in leading edge shape such as nose droop which prevent this will also increase the region in which relatively low lift-dependent drag may be expected. Thus though the zero-lift drag of a wing having a



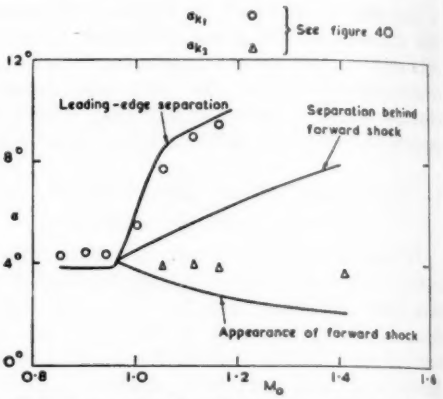


FIGURE 41. Relationship between kink incidences and flow boundaries for cropped delta half-wing. The outboard shock does not appear on this wing below a Mach number of 1·4.

FIGURE 40 (*left*). Simple straight-line representation of lift-dependent drag curves.

drooped section may be higher than that for a similar symmetrical section, the reverse may be true at a lift coefficient of above, say, 0·2. Wind tunnel tests on untapered wings of 50° sweepback show that at subsonic and transonic speeds the gains from a drooped section operating at moderate lift can be appreciable. This effect diminishes rapidly as the stream Mach number exceeds about 1·1 and almost disappears at a Mach number of about 1·2.

6. Concluding Remarks

In the preceding pages a brief survey has been given of some of the phenomena and problems which have arisen during tests at the N.P.L. on a number of wings having leading edge sweeps near 50°, and without twist or spanwise change in profile. The information presented has therefore been obtained from a limited class of wing. Moreover it may be argued that many of the flow features considered would not appear on wings which have been properly designed, using the many artifices at present available to the high-speed aerodynamicist. While there is much substance in these views they are not overwhelming. The general flow development discussed in Sections 2 and 3 is applicable to wings of lower, or even higher, leading edge sweep, though naturally there is some shift in emphasis, particularly of the relative magnitude of the flow regimes dominated by leading edge separation, or by attached leading edge flow and shock waves. The fundamental unity of all transonic wing flows remains, however. Obviously further research is required to obtain a more complete knowledge of the general subject, to determine for example the influence of sweepback on transonic flow attachment, or the effect that changes in wing profile have on the details of the flow development.

Once the basic structure of the flow has been understood in some detail, it should become possible to design wing shapes to avoid undesirable features such as leading edge separation, or the formation of an outboard shock. The beneficial effects of body contour, wing twist, or curved-tip plan forms should be more easily assessable especially in those flow conditions not readily examined by existing theory; new approaches to the attainment of shock-free transonic wing flow may become apparent. The ultimate object is the optimum wing design for any given task and this must always be pursued, since it is vital for the efficient operation of transonic or supersonic transport aircraft. And since this optimum performance by the very stringency of the aerodynamic design may well be obtainable in only a limited range of wing lift and flight Mach number, a proper understanding of the possible flow phenomena which may develop in off-design conditions is essential.

The limitations of the data described in the present paper, though important, are therefore perhaps of less magnitude than might at first be supposed. Moreover this work is but a part of the large and co-ordinated assault being made in this country on the complex, yet fascinating problems which arise with swept wings in transonic air flows.

ACKNOWLEDGMENT

The work described in this paper is part of the research programme of the National Physical Laboratory and the authors acknowledge the permission of the Director of the Laboratory to publish it.

Sir Hubert Wilkins, Father of Arctic Aviation

by

JOHN GRIERSON

(de Havilland Engine Co. Ltd.)

AS MAN'S CONQUEST of the air was gathering momentum in the 1920s, there came a longing among explorers of the Arctic, to overfly the icy waters which had hitherto taken such a heavy toll in men and ships. Men who, like Wilkins, had already made their mark in sailing through the ice and walking over the eternal snows, looked to the aeroplane or airship as a wonderful observation platform and as a means of quicker travel. And of the great of many nations who tried their skill and courage in the early attempts to fly across the Arctic, George Hubert Wilkins stands out as greatest of them all.

Arctic Rivalry

Earliest airman of the Arctic had been Andrée, the brave Swedish professor who set out in a balloon with two companions from Spitzbergen in 1897, hoping to be drifted across the Polar Basin. That attempt was short-lived, ending in complete disaster and the eventual death of all the crew. In 1925, Roald Amundsen, the Norwegian who had triumphed by surface transport both over the North West Passage and the South Pole, set forth from Spitzbergen with the American Lincoln-Ellsworth and the Norwegian Riiser-Larsen on a flight to the North Pole, using two Dornier flying boats. One machine was forced down within 200 miles of the Pole by engine trouble, and the other went to its rescue. There followed a protracted and ghastly struggle, against ice conditions which can scarcely be imagined, to take-off the good flying boat in a bid to save the lives of the expedition. Over three weeks elapsed before the starving men, almost at their last gasp, completed a crude ice runway and took off successfully. After this escape, Amundsen and Ellsworth and Riiser-Larsen decided to try using an airship and engaged the Italian designer Nobile to pilot them across the Pole in his dirigible, the Norge. In this they were successful and landed safely at Teller in Alaska after a flight of 71 hours. Just before the Norge left Spitzbergen, Commander Byrd had arrived, assembled his trimotor Fokker and, on 9th May, 1926 made a brilliant flight to the Pole and back in 15½ hours.

These, then, were the great names contemporary with Wilkins in the early days of Arctic flying—Andrée, Amundsen, Ellsworth, Riiser-Larsen, Nobile (whose subsequent attempt at another Polar flight ended in disaster), and Byrd. Others indeed there were, such as the Dane Lauge Koch, who did some of the earliest air survey work in Greenland, and Gino Watkins leading the British Arctic Air Route Expedition, but their work was scientific rather than operational.

Received March 1960

Courtesy of Lady Wilkins

Sir Hubert Wilkins standing by a portrait, painted by his wife, of himself and Ben Eielson in front of the Lockheed Vega in which they flew from Barrow to Spitzbergen in April 1928.

Wilkins' Background

Wilkins' prowess in the Arctic was based on preparation, persistence, and achievement, co-ordinated to a unique degree. This Australian farmer's boy, lacking a scholastic background save of the most rudimentary nature, became interested in meteorology at an early age because of the appalling effects of three years' drought on his father's farm in South Australia. He believed that if only long-range forecasts could be developed, the effects of localised droughts such as afflict the Australian Continent, could be greatly lessened by temporarily transferring herds of cattle until the danger was passed. Study of the movement of air masses would have to include what happened over the Polar Regions, of which little was then known, and this was the underlying motive of all Wilkins' work, both in the Arctic and Antarctic.

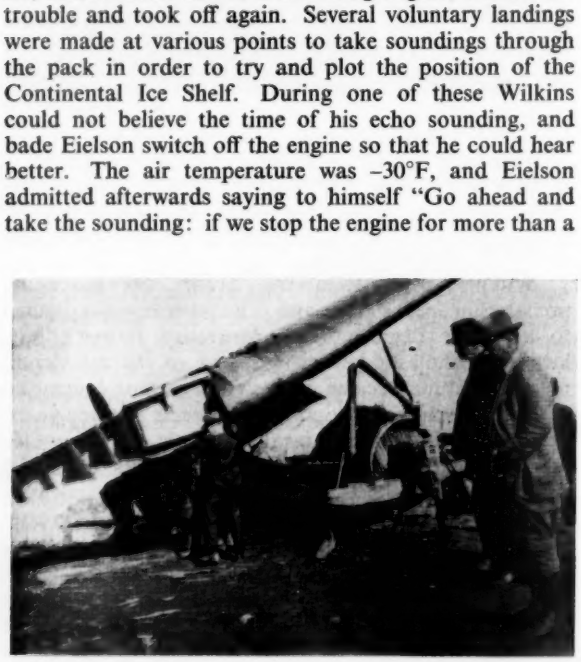
Wilkins served his apprenticeship with Stefansson in

Alaska and the Canadian Arctic where he went for three years in 1913, officially as photographer for Gaumont, but in point of fact he was such a good all-rounder that his leader described him as "easily my best man" and on one occasion he saved Stefansson's life.

That was without aeroplanes, but in 1926, after serving with Shackleton in the Antarctic and leading a two years' tropical expedition to N. Australia, Wilkins returned to Alaska as leader of an air expedition sponsored by the North American Newspaper Alliance. The first object was to explore the area beyond Point Barrow, for no one had ever been more than 50 miles due north of there. Two Fokkers, one with a single Liberty engine and the other a tri-motor, were assembled at Fairbanks where the temperature was –52°C. Seldom has an expedition started on such a disastrous note, for within 24 hours both aircraft had been crashed on landing after their first test-flights, and a newspaper reporter had been killed through walking into one of the propellers. Both crashes were probably due to optical errors in approaching a runway 40 yds. wide between two banks of snow. Nothing daunted, Wilkins set to to repair the smaller aeroplane, and used it to fly petrol in drums 550 miles across the unsurveyed Alaskan mountains up to Point Barrow in preparation for operations thence the following year. That he succeeded in building up a cache at this range, with such a small aeroplane, was remarkable.

Landings Far Out on the Arctic Ocean

In the next year's operations, a couple of Stinson ski-equipped biplanes, which were specially suited for landing on the pack-ice, were used. Wilkins, with his pilot Ben Eielson, now flew out 500 miles over the pack-ice, force-landed due to a mis-firing engine, cleared the trouble and took off again. Several voluntary landings were made at various points to take soundings through the pack in order to try and plot the position of the Continental Ice Shelf. During one of these Wilkins could not believe the time of his echo sounding, and bade Eielson switch off the engine so that he could hear better. The air temperature was –30°F, and Eielson admitted afterwards saying to himself "Go ahead and take the sounding: if we stop the engine for more than a few minutes, we will never get it started again, and nobody but God and you and I will ever know what that sounding is". This was typical of Wilkins' utter disregard of his own safety.



The crash of the Liberty-engined Fokker after its first test flight at Fairbanks in 1926. From *Under the North Pole*, 1931.

George Wilkins, as he was then, working a cinema for the benefit of an Eskimo audience in Alaska, in 1914. (From *Under the North Pole* by G. H. Wilkins. Published by Benn, 1931.)

Because of the time spent in repairing the engine and the landings for taking soundings, daylight was running short for the return flight home. To make matters worse, they struck a headwind and in the ensuing darkness, accompanied by a blizzard, ran out of fuel 65 miles from Barrow. By the most amazing luck, their "dead-stick" glide from 4,000 ft. hurled them into a snowdrift, smashing the undercarriage and lower wing, but brought their aeroplane to rest on the only bit of smooth ice for miles around. At daylight they discovered by soundings that their floe was moving N.E. at 5–6 m.p.h. and for four days they were confined to the machine by a gale. All the time they were being drifted farther away from land. There then began the desperate trek for safety.

Wilkins built ice-houses for shelter each night single-handed because Eielson had had his hand frost-bitten. It was only Wilkins' experience with Stefansson that brought them through alive after thirteen days with precious little food, sometimes stumbling over old floes and pressure ridges, sometimes falling into ice-cold water. Never before had anyone survived landing far out on the Arctic ice, either because their engine was failing or they wanted to measure the depth of the ocean.

Having established that there was no land to the North of Barrow (the mythical "Keenan Land" was evidently due to enormous pressure ridges, looking like land) Wilkins next wanted to explore in a North Easterly direction towards Etah (near Thule) in Greenland, since there were considerable doubts concerning the full extent of the intervening Canadian Arctic islands. However, the season was too late that year.

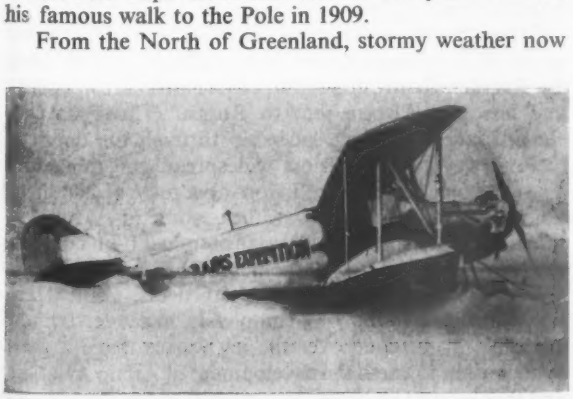
Across the Polar Basin

Wilkins and Eielson returned for the third year running in 1928 with the newest and most efficient aeroplane then made—a Lockheed Vega. Taking off on

a ski undercarriage, they planned to fly over the Canadian Arctic Islands and then around the North of Greenland to Spitzbergen, thus completing a circle of more than 2,000 miles over the Polar Basin. This was to be the flight towards which the work of the previous two years served as preliminaries, and Wilkins patiently awaited at Barrow the spell of fine weather which records indicated usually followed a period of stormy weather in late March. The intention was to navigate mainly on sun-shots taken with an R.A.F. Mk. V bubble sextant. Since the average rate of variation change was expected to be something like ½° per minute on a flight of 20 hours, this meant that the magnetic compass would be of limited value, apart from taking a long time to settle on the part of their route (which was substantial) near to the Magnetic Pole. But after the 5th of April, the sun would be above the horizon all the way to Spitzbergen, and that would be ideal for solar navigation.

On the first attempt to take off on a cross-wind strip prepared by the Eskimos, the Lockheed bounced off the runway and broke one of the metal skis. Luckily they had a spare pair of wooden ones which were now fitted. A new runway had to be made for the heavily laden aeroplane and it was not until 16th April that they were able to make their start. Over the pack they flew against a slight headwind which reduced their ground speed to 108 m.p.h. Soon the ice was broken by leads of open water, and from the jagged ice scenery below, Wilkins was able to read the direction of pressure, caused by wind and tide. The wind went round to South West, but near Latitude 80° North, where they particularly hoped to see if there was any uncharted land, a sheet of cloud covered whatever lay below. This lasted until 120 miles farther on, when they found they were over old heavy pack. From 3,000 ft. no sign of land could be seen. North of Grant Land they altered course to fly round the rim of a storm and at midnight (local time) and 11 hours after take-off, the dull red orb of the sun was well above the horizon to the North. After ploughing through several banks of clouds, suddenly they broke into the clear and saw the glaciated mountains of Grant Land barely 20 miles away to starboard. This coincided exactly with their estimate, which gave the two airmen a great feeling of confidence in their navigation. They flew almost over Cape Columbia whence Peary had started his famous walk to the Pole in 1909.

From the North of Greenland, stormy weather now lay ahead in the direction of Spitzbergen, but Wilkins and Eielson were in complete agreement that it would be safer to carry on than to attempt an immediate forced-landing on the ice. The outside air was –48°F. and they knew that there would be violent mixing with the warmer air around the land-mass of Spitzbergen. After climbing to 8,000 ft. for a sun-shot they had to let down into a boiling cauldron of cumulus cloud, in extremely turbulent air. The compass, in the weak magnetic field, had hardly time to settle between the disturbances. As they neared the sea, loose objects were thrown around the cockpit by the bumps, for a blizzard was raging. Hurriedly, Eielson picked the first patch of smooth snow he could find through his oil-covered windscreen and lobbed down. They could not see a thing, but Wilkins managed to jump out and drain the engine oil before it froze. The gale raged, the machine shook as the wind tugged at their wings. Hours later, after both men had been to sleep, the weather moderated and they were able to take a shot of the sun. This put them between Kings Bay and Green Harbour, but before they could survey their position properly, the blizzard started again.



One of the Stinson Detroiters abandoned on the Arctic Ocean in 1927. From *Under the North Pole*.

For four whole days they were pinned down and then when the clearance came, the machine kept freezing solid onto the snow and would not budge until Wilkins pushed it. Twice he failed to scramble aboard and Eielson took off without him. On one attempt Wilkins grabbed a rope dangling from the cockpit with his mouth, and loosened all his teeth, hitting the tail-plane as he fell off into the snow. Only by using a piece of driftwood as a sort of punt-pole did Wilkins finally manage to start the machine and stay on board.

As soon as they had climbed to 3,000 ft. they were able to make out the radio masts of Green Harbour and there they landed safely at last. The site of their forced landing, they learnt, had been inappropriately named Deadman's Island.

This flight of 2,500 miles (2,200 miles from point-to-point) in just over 20 hours' flying was the first ever to be made by an aeroplane across the Polar Basin—in fact only Amundsen's airship had hitherto made a similar kind of voyage. Little wonder therefore that when Wilkins visited London, he was accorded the honour of Knighthood by King George V in recognition of his outstanding work in the Arctic, and from the Royal Geographical Society he received the Patron's medal "for his many years systematic work in Polar regions, culminating in his remarkable flight from Point Barrow to Spitzbergen". Amundsen described this achievement in the following words "No flight has been made anywhere, at any time, which could be compared with it".

From the foregoing alone, Wilkins stands out as the Father of Arctic Aviation. But that is by no means the end of the story. In the same year that he was knighted, Wilkins sailed South with the same aeroplane and the trusted Ben Eielson as pilot, to make the first flight ever in Antarctica. He did not return to the Arctic until 1931 when he went there in a submarine called *Nautilus* on an attempt to reach the Pole. Although this venture was a failure because of the very poor condition of his submarine, Wilkins had the *idea* which he lived to see

successfully executed in the Atomic Age. This lapse from aeroplanes to submarines was only temporary, for nine years after his great achievement in the Arctic, Hubert Wilkins returned North as leader of the most remarkable air rescue operation ever made in the Far North.

The Levanevsky Search

In August 1937, the Supreme Soviet had commanded Sigmund Levanevsky, accompanied by five other comrades, to set out in a four-engined Tupolev on a flight from Moscow across the Pole to Fairbanks, Alaska. All seemed to go well at first until, just after passing over the Pole, radio messages came through to say that they had engine trouble. The reception was faint and no details could be understood. Silence followed. Levanevsky never reached Fairbanks and his aircraft was posted as missing. As soon as Wilkins heard the news, he rang up the Soviet Embassy in Washington, with the offer of leading an immediate search towards the Pole. This was gratefully accepted by the Russians, who gave him authority to find an aeroplane and crew right away. Within four days Wilkins had found and bought a Catalina long-range flying-boat, and had fortunately secured the services of Hollick-Kenyon, the famous Canadian who had piloted Ellsworth across Antarctica in 1935. They planned to search a strip 1,200 miles long from the Pole and 400 miles wide, starting right away, although August is one of the worst months for fog and rotten honeycombed ice in the Arctic. Two days later, the Catalina went off to Coppermine on an expedition which would ordinarily have needed six months' planning and preparation. Wilkins insisted that only an early rescue could be a successful one and he drove everyone unmercifully. But how he managed to get away in six days, no one will ever know.

On the hop to Coppermine, weather trouble began immediately with heavy icing in the clouds and fog down below. Yet the search began in earnest on 22nd August, when the cabin was so full of leaking petrol drums that Wilkins was almost asphyxiated by the fumes from the bilges. Nevertheless, on this first occasion they flew up to 78°N before being forced back by bad weather. And so they scanned the sea and the Canadian Arctic islands in perpetual daylight; calls for Levanevsky were transmitted at intervals but in spite of a continuous watch, there came no reply. The first flight lasted 13¾ hours, but much longer ones were extended by landing in some rocky cove in order to refuel from the drums. Difficulty was of course found in steering accurate courses, due to the nearness of the Magnetic Pole in most of this area. Kenyon often had to rely on dead-reckoning, oriented from observations of drift, direction of snowbanks, pressure ridges, lanes of open water and other objects.

A gratifying step in international relationships was an agreement whereby special reports of Polar weather were supplied by Washington as a result of a tie-up between the meteorological offices of the U.S.A., Canada, Norway, Sweden, Denmark and of course, the U.S.S.R. In spite of this, the Catalina had to operate at times in unexpectedly adverse conditions, such as when both its air speed indicators were frozen up, and weather at base was deteriorating while they were within 350 miles of the Pole.

"Planet News"

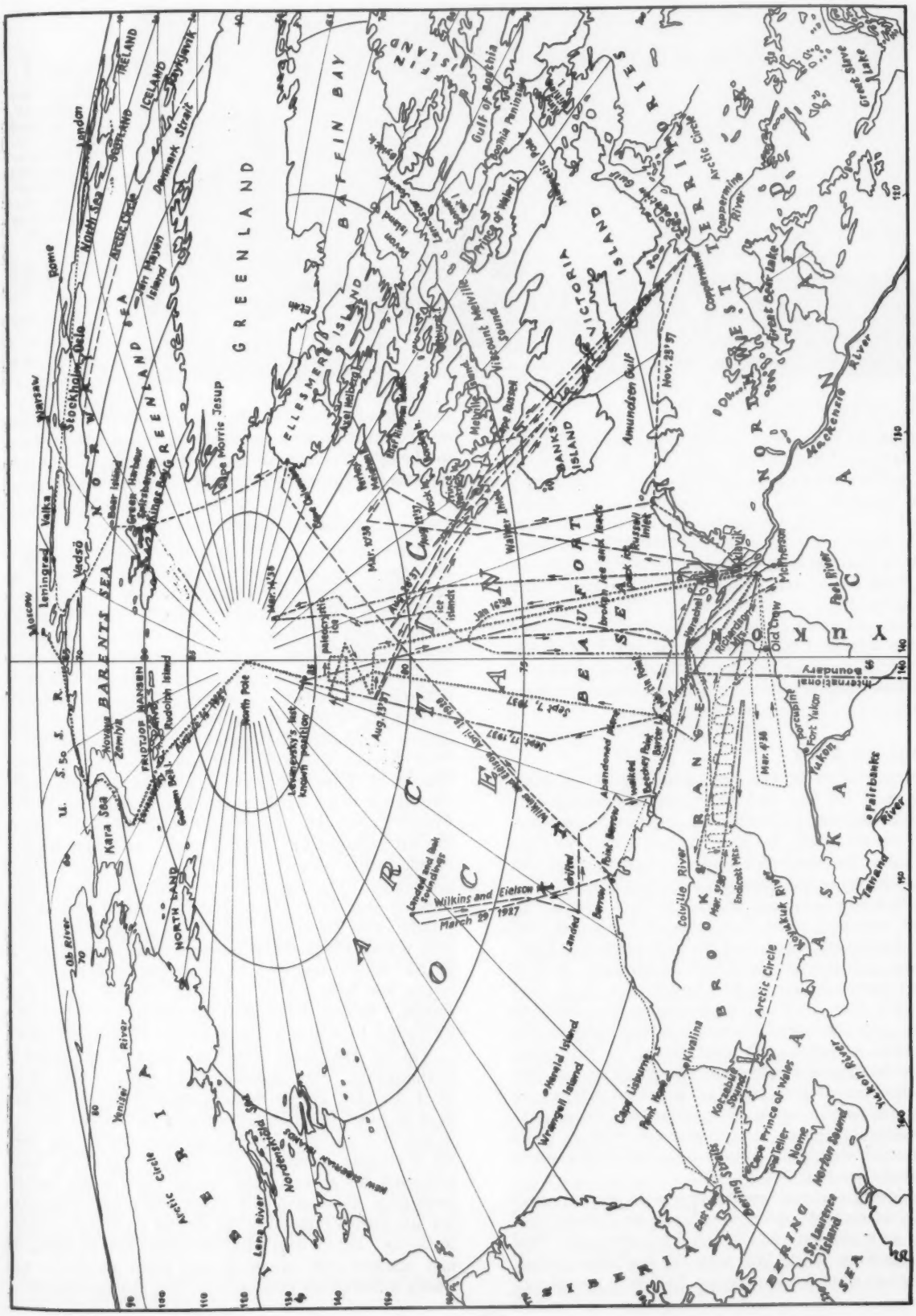
Wilkins, Hollick-Kenyon and Limbaermer in the hatch of the Catalina flying-boat at the start of the search for Levanevsky.

For a whole month, the search continued whenever the weather was even slightly favourable, but towards the end of September, conditions were becoming so bad that they had to return to New York.

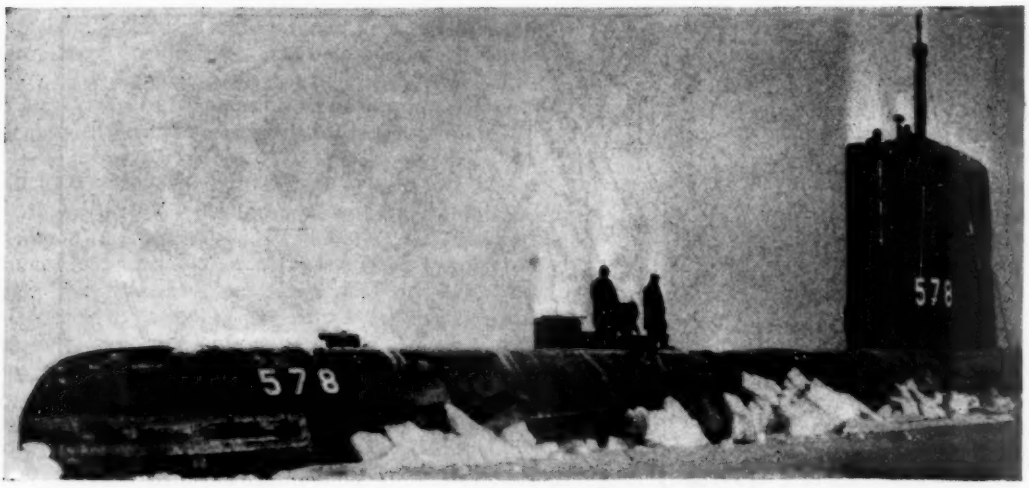
The Soviet Embassy now bought Wilkins a ski-equipped Lockheed Electra, fitted with long-range tanks, so that he could continue the search by moonlight in the winter. There were good grounds for believing that on a background of snow and ice, shapes and shadows might show up better than in broad daylight. In fact the heavy shadows cast by the moon proved to be even blacker than those of the sun.

November saw the beginning of the second phase of the search, from Edmonton. Flying from Aklavik, Barrow and Old Crow, the expedition combed the Alaskan Mountains which seemed the most southerly area that Levanevsky could have reached, but all without sighting a single clue. On one take-off for an eight hour flight over the ice from Aklavik, the propeller of the port engine struck a ridge and was bent without the crew realising it at the time. Later, on the ground luckily, the engine seized solid, but the Soviet Embassy organised the supply of a replacement engine which was flown up and fitted within ten days.

These flights, covering an extensive area of the Arctic up to 85°N continued until the middle of March when the Russians finally decided to call a halt to the operation and have the Electra sent to Russia. Thus was concluded under Wilkins' leadership through the depth of the Arctic winter, the most widespread and dangerous air search ever attempted, involving a total of 284 flying hours, often in adverse weather, sometimes by day, sometimes by night. The Russians too had been combing the region on their side of the Pole. They had used more aeroplanes and had lost six in the process. As a rescue operation, the result was completely negative, but the extensive flying done over the icy wastes made a great contribution towards the development of Arctic Aviation, and thus in itself became a fitting memorial to Levanevsky and his five companions.



The Arctic Ocean, showing Wilkins' major flights from 1927-38, and the course of Levanevsky from Moscow.



The U.S. Navy Submarine, *Skate*, surfaced at the Pole for the ceremony of committing Sir Hubert Wilkins' ashes to the four winds.

(Courtesy of U.S. Navy. A U.S. Navy Official Photograph.)

Wilkins' Other Work

During the Hitler War, Wilkins was appointed Arctic Consultant to the Quartermaster General of the U.S. Army, and immediately introduced far-reaching changes in the Arctic clothing and equipment of the Army. He made a survey of the Aleutians in search of air bases and travelled extensively in the Far North. But although he flew to many military centres, such as Churchill and Thule, both during and after the war, his aviation these days was no longer of the exploratory kind. All Wilkins' work now, and in the 18 years during which he remained as adviser with the U.S. Army until the day of his death, consisted of applying his tremendous background of "know-how" and experience to the present-day problem of maintaining U.S. air power in the Arctic.

Wilkins the Man

George Hubert Wilkins was a quiet and astonishingly modest man. His face, with his impish smile, always bore the marks of frostbite he had received in Stefansson's expedition. In speech, his powerful and melodious voice had neither an American nor an Australian accent, although he remained throughout an Australian citizen.

He had two outstanding characteristics. Firstly his courage, springing from deep religious conviction, and making him entirely fearless of his own safety. Secondly, his vivid and exceedingly fertile imagination, divided into two parts, one very good about greater things, and the other a complete blank in visualising the possibility of things going wrong. This very combination gave him tremendous strength, and contributed to his determination to succeed in situations where any normal man would have accepted defeat. Easy to get on with, a man of sparkle, with a sense of humour and bearing an unshakeable loyalty to his companions, Wilkins could inspire confidence in his followers and lead them where he wanted them to go.

Arctic and Antarctic exploration were only a part of Wilkins' exceedingly active life. Early he distinguished himself as a photographer in the Turco-Bulgarian War and in the First World War with the Australian forces, when he was wounded nine times; he flew in aeroplanes, balloons and airships. He was an excellent naturalist and got on well with cannibals, besides experimenting in thought transference over 3,000 miles, on which he wrote a book and contributed to another about religion.

Legacy of the Arctic

The realisation of Wilkins' work in the Arctic, and of the others who expanded and developed his ideas, can be seen in the regular scheduled airline services which today span the Arctic from Europe to North America, to Japan and to Australia. S.A.S. were the first in the field with their route from Copenhagen to Los Angeles by way of Sondre Stromfjord (Greenland) and Winnipeg, followed by a second spur to Tokio by way of Anchorage in Alaska. Canadian Pacific Airlines were the first British Commonwealth operators to enter the field, and their service, which has to terminate at Amsterdam instead of London for political reasons, flies across Greenland and Canada to Honolulu and Sidney. Air France Constellations go from Paris to Tokio by way of the Arctic, and Pan American spans the San Francisco-London route on a great circle which takes them across Greenland. For administrative reasons T.W.A. temporarily suspended operating between Los Angeles and London over Greenland but started again in May 1960. Luft Hansa will probably be the next to take advantage of the better weather and shortened distance of the Arctic and later our own B.O.A.C., which has always shown a distant enthusiasm for the frozen North, will be bound to follow in the footsteps of its competitors.

On 1st December, 1958, the maid in the hotel at Framingham, near Boston, Massachusetts, where he lived, found Sir Hubert Wilkins dead on his bed. He had died of heart failure after working on his car for two hours in a car park on the coldest day of the year. As a striking tribute to his memory, the U.S. Navy conveyed his ashes to the North Pole in the nuclear submarine *Skate*, and in a simple ceremony committed the last earthly remains of the Father of Arctic Aviation to the ice: the ice over which he had flown, on which he had crashed and struggled, and from which he had finally achieved his greatest and most lasting triumph.

Engineering Aspects in Man Powered Flight*

by

B. S. SHENSTONE, M.A.Sc., F.R.Ae.S., A.F.I.A.S., F.C.A.I.

(Chief Engineer, British European Airways)

1. Introduction

There are three practical aspects to the problem of man powered flight: aerodynamics, physiology and engineering. There are many more impracticable aspects to man powered flight, including the philosophical, the fanatical and the sceptical, which unfortunately have not yet been included in the lectures of the Man Powered Aircraft Group but which will no doubt find their places.

The engineering aspects are fortunately fairly clearly defined, although there is likely to be a slight threat of interference with the aerodynamic problems. I consider the engineering problems to be these: —

(*i*) the arrangement of the structure,
(*ii*) the details of the structure,
(*iii*) power transmission.

In all three of these one has aerodynamic limitations which may be illustrated in this way. The arrangement of the aircraft must be structurally sound and yet must be influenced by the aerodynamic requirements such as those of drag and interference. The details of the structure with regard to such matters as the wing thickness and the surface smoothness are limited by aerodynamic requirements. As for the power transmission, it may be influenced decisively by the aerodynamic requirements of the propeller position, its size and its speed of rotation.

It will already have been noticed by those interested in flapping flight that I have mentioned a propeller in such a manner as to exclude the use of flapping wings. This is done for reasons which have been expressed before over the past few years, and I think it is worth while mentioning them now so that my standpoint will be as clear as possible. I propose the propeller drive for the first generation of man powered aircraft because it is the easiest and best-known drive to adopt. This design brings us hardly outside our present knowledge. Practically all the factors can be calculated and those that cannot be exactly calculated can easily be tested in a wind tunnel or in the open air. Efficiencies are known and weights can be calculated. What more can one ask?

Although some people have reason to say that the flapping wing might be marginally more efficient than the propeller, it is unfortunately unproved at anything over 10 ft. span, even when driven by engine power. The complexity needed to enable the wing incidence to change in the proper way during flapping motion is complex and the weight of such full-scale mechanism is unknown. The aerodynamics of the rapidly oscillating tip are not known sufficiently so that one can calculate the performance of the flapping wing with any accuracy.

It is admitted that a natural rowing motion may be of the right order for easy transmission to a flapping wing, but even here the mechanism at this stage of development is far from simple. Therefore, without making any attempt to denigrate the future of the flapping wing, I would certainly say that it is a future development, not an immediate one. I may be criticised for looking for quick results, but that is my object; to get the men flying as quickly as possible, even though it may not be at the ultimate efficiency.

NOTATION

A	aspect ratio $= b^2/S$
b	span (ft.)
C_D	drag coefficient
C_{Dp}	profile drag coefficient
C_L	lift coefficient
D	drag (lb.)
L	lift (lb.)
P	power in horse power
S	wing area (ft.^{2})
V	air speed (ft./sec.)
V_s	sinking speed (ft./sec.)
w	wing loading (lb./ft.^{2})
W	weight (lb.)
W_e	empty weight (lb.)
W_w	wing weight (lb.)
ρ	air density $= 0{\cdot}00273$ (slug/cu. ft.)

2. Arrangement of the Structure

The structural and aerodynamic calculations indicate that the first man powered aircraft would have a wing of high aspect ratio, that is, of the order of 15. This is not an exceptionally high aspect ratio because cantilever wings have been built with an aspect ratio of 30[1]. It is also evident that the wing loading of man powered aircraft should be within the range of 1–3 lb./ft.^{2} This indicates roughly the size of the wing, which, depending on whether it is a single- or two-seater, will have an area between roughly 75 ft.^{2} and 300 ft.^{2}, with a span between 35 ft. and 65 ft. A large wing may be difficult to handle at low speeds, but if it were unfortunately necessary to have one, it must be accepted.

It is still an open question whether such a wing should be low on the fuselage or high. Aerodynamically it is far easier to deal with the high wing arrangement when considering the problem of interference and root stall at the high angles of flight which would be normal. On the other hand, a low wing would be closer to the ground and, if the ground cushion effect is to be

*The second lecture given to the Man Powered Aircraft Group —on 29th January 1960.

important, the closer the better. Whether a low wing arrangement could be devised with acceptable high angle of attack and flying characteristics is an important point which is as yet unsolved.

If we assume that the aircraft will fly at 10 ft. altitude, that is, 10 ft. from the bottom-most point, a low wing would be something like 4 ft. closer to the ground than a high wing, which would be of enormous importance if its effective aspect ratio can be maintained at the highest possible figure. On the other hand, most of the projects made public to date have been high-wing types because they are aerodynamically easier and, with wings which will tend to be very flexible in bending, they are less liable to damage on take-off and landing.

Leaving the wing for a moment, it is necessary to think of the tail/propeller combination as a unit. The first thing is that the propeller should not be in such a position as to disturb the free air flow over the wing, which in practice means it should be located behind the wing as a pusher-type propeller. This means it could be behind the tail of the aircraft, or it could be at the stern of a tail-less aircraft or a tail-first aircraft, or between the booms of a twin-boom aircraft, or above or below the boom of a single-boom aircraft, or by some ingenious means be incorporated in what might externally look like a normal fuselage.

It is essential that any layout must enable the propeller to be sufficiently large in diameter so that the best possible efficiency may be achieved. This diameter is bound to be over 6 ft. and may be as great as 9 ft. This large diameter member may well determine the fundamental layout of the aircraft.

As for the main structural material, I consider that for prototype aircraft, some form of wood construction would be the optimum. Whether or not wood in certain places is stabilised by honeycombs or foam plastic does not rule out wood as the main strength member. There may well be better materials than wood, but if anybody is starting now to design an aircraft which is meant to be a serious flying machine, he will not be able to afford fundamental research into more exotic materials. He can afford to use only materials which have proved themselves in the past. It is likely to be discovered that light-weight wood, that is, high-quality spruce and other woods even lighter, such as balsa, may take leading parts in the structure.

With wing chords up to 3 ft. and down to 1 ft., it will be realised that the rib loads at speeds of the order of 30 m.p.h. and an ultimate load factor of $2\frac{1}{2}$ will be very small indeed, and to get local structural stability a bulky structural material will be necessary.

It will also be clear that the lightest possible covering will be necessary with the proviso that the aerodynamic requirements of surface texture and continuity will have to be observed.

Perkins' use of an inflated wing design should not be ignored, but it requires special experience not generally available[2].

This leads one to the fuselage structure which, although it will have to have strong points involved with the wing attachment, pilots' weights, mechanical driving loads and landing and take-off loads, will need an aerodynamic fairing which must be as light as possible. This may inevitably lead to a light fabric covering. It is known that it is impossible to get a pure fabric covering to take an optimum aerodynamic form, but it may be necessary for considerable wind tunnel testing to be undertaken to achieve the optimum form for the designed cruising speed with the limitations on shapes achievable with fabric.

The arrangement of the structure is also influenced by the type of undercarriage, which again depends on the type of take-off visualised. The current condition laid down for the £5,000 Kremer prize is that the aircraft would have to take off without power storage and without outside assistance[3]. This probably means that during a large portion of the take-off, the drive will have to be through wheels in contact with the ground which has aerodynamic and structural complications apart from mechanical problems. If the take-off is accomplished by stored energy or by external power, the undercarriage need only be a skid with rubber buffers, which is a much simpler scheme.

3. The Details of the Structure

It is the details of the structure which weigh the most. The primary units, such as the wing spar and the fuselage skeleton, may be very light indeed, but the weight goes in wrapping these primary structures into an acceptable aerodynamic shape. If the load factors and the aerodynamic loads are large enough, it is quite easy to devise a thick-skinned wing which does not wrinkle under normal flying loads. In powered aircraft this can nowadays be accomplished by using metal skins and in sailplanes by using light-weight, fairly thick plywood, or sandwiches, because the ultimate factors and speeds are quite high (typically ultimate factor of about 8 and diving speeds of about 120 m.p.h.), but for the man powered aircraft, the ultimate factor may be only 2·5 and the speed to be catered for only about 50 m.p.h. (Fig. 1). Such low figures theoretically result in a very light aircraft, as indeed they could in practice, were it not for the aerodynamic difficulties, mainly connected with wing surface shape.

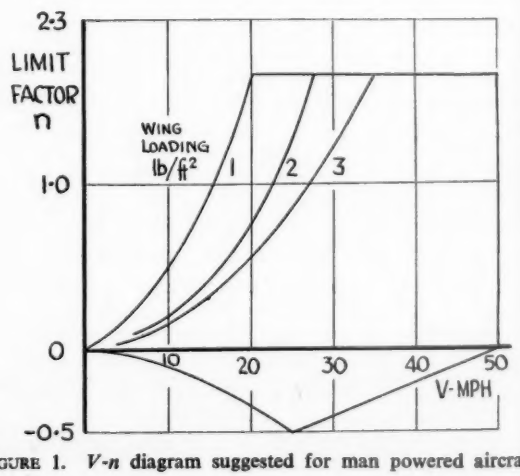


FIGURE 1. *V-n* diagram suggested for man powered aircraft.

At Queen's University, Belfast, a number of schemes have been studied and at the moment they feel it is most worth while using a single spar with a plywood "D"-type nose with fabric covering over the rear part of the wing which is the classical sailplane structure. They do not yet know the best way of stabilising the leading edge plywood.

We all have seen or read about new schemes for stabilised skins and much success has been achieved in using balsa-backed thin plywood, paper honeycomb backed thin plywood or glass fibre and many other variations of this scheme. Some schemes even go so far as to fill the whole interior of the wing with a light foam plastic or coarse paper honeycomb. Fig. 2 shows an impregnated paper honeycomb developed for a sailplane in Darmstadt.

There is no doubt whatever that a satisfactory aerodynamic surface could be obtained by a number of these methods. However, they vary widely in cost and difficulty in manufacture and also, in durability and mainly in weight.

It is very important to study the interaction between weight and drag in order to design the aircraft so that it will require the minimum amount of power for flight. There are many ways of expressing the power required and one of the simplest ways is as follows: —

$$P = \frac{W}{L/D}\,\frac{V}{550}\,.$$

It will be seen from this that one wants a low weight, low speed and a very high L/D or good gliding angle. This must be fairly obvious and it may be better to put the same formula into another shape, such as,

$$P = \frac{W}{550}\sqrt{\left(\frac{2W}{\rho S}\right)}\cdot\frac{C_D}{C_L^{3/2}}$$

(where $\sqrt{\left(\frac{2W}{S}\right)}\cdot\frac{C_D}{C_L^{3/2}}$ is the sinking speed)

and then $P = \frac{WV_s}{550}$.

It will be seen from this that the wing loading (W/S) should be low and that, quite apart from this, the all-up-weight should be low. It is also clear that the sinking speed must be as low as possible, which requires a low drag coefficient and a high lift coefficient. This expression shows clearly the extra difficulty of the man powered aircraft problem compared to the sailplane. Sailplane weight may be allowed to increase considerably and still achieve a better sinking speed. But for the man powered aircraft we have W the weight as an extra positive factor warning us clearly about the value of weight reduction in improving the power required. Here we see the usual inconsistent requirements of any weight versus drag problem, and it must be obvious, for instance, that to minimise the weight one dare not make the wing loading too low or the weight will begin to rise because of much larger wing. It was also easy to overdo the effort in getting the drag coefficient as low as possible because by doing so one can add weight, which would at least eventually counteract the effect of reducing the drag. A well-known analysis of reasonable approximation was that the minimum power required occurs when

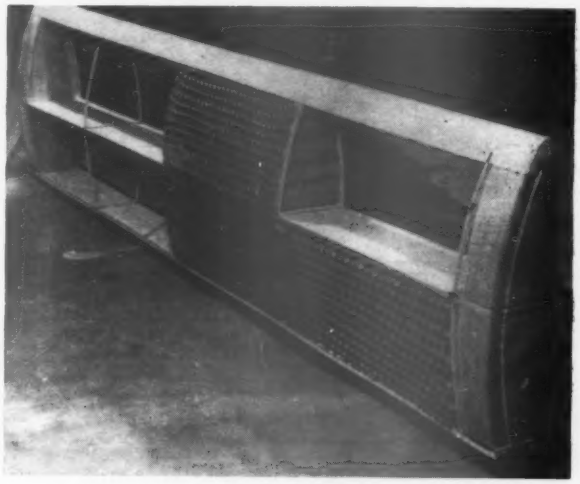


FIGURE 2. Impregnated paper honeycomb for sailplane wing (Darmstadt).

$$C_{Di} = 3C_{Dp}$$

This means that, of the total power required, one quarter is needed to drive the aircraft and three quarters are required to sustain it.

This power for sustaining the aircraft is often called the induced power. This induced power may be expressed approximately as: —

$$P_i = \frac{W}{60A}\sqrt{(C_L w)} \text{ at sea level.}$$

and this expression instructs us to do some obvious things and some impossible things. We must keep the weight down. We must use a large wing aspect ratio. It tells us also, although in a somewhat more minor key, to use a low wing loading and to fly at a low value of the lift coefficient. At least this does show us how we might proceed in playing with weight and drag, but before we can do much about it, we have to know more about the weight problem.

The structure weights to be expected for properly designed man powered aircraft are not easy to foresee because there are so few data available on which to base them. The only information which is factual is the meagre information on the empty weights and some other weights of five single-seat man powered aircraft actually built, but none of them completely successful. Wing loadings come to an average of about 2 lb./ft.^{2} and aspect ratios average about 15, the ultimate load factors being something of the order of 5 or 6, but full details are lacking. If we plot the available empty weights against span, and compare them with sailplane empty weights, it seems reasonable to assume that the

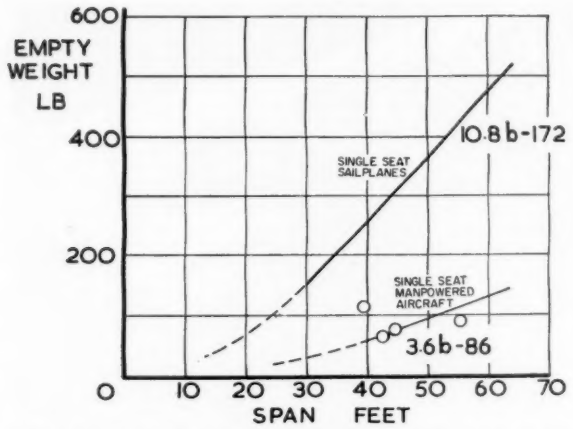


FIGURE 3. Empty weights plotted against span for single-seat sailplanes and single-seat man powered aircraft.

curve will be linear between the useful spans of from 40 ft. to 60 ft. From this we find that for such a single-seater:

$$W_e = 3{\cdot}6b - 86 \text{ in pounds and feet.}$$

However, this information, shown in Fig. 3, does not take us very far, because we want to have more information about these aircraft than just the empty weights and somehow we have to have some line on the variation of wing weight with span. There are not enough data available and we have to fall back on actual sailplane weights which, although of a slightly different order, are as near as we can get to our problem. Fortunately the publication of *The World's Sailplanes* by OSTIV in 1958 made it possible to analyse sailplane weights for the first time, and this has been done by Piero Morelli in an OSTIV paper[4]. The following notes are based on Morelli's analysis, although the conclusions which I have made have not been agreed by him.

In an attempt to obtain wing weights, we find that for single-seaters, based on 24 different types:

$$W_w/W_e = 0{\cdot}215 + 0{\cdot}0077b.$$

If we assume that this ratio based on sailplanes is applicable to man powered aircraft, we can then say that:

$$W_w = 0{\cdot}0277b^2 + 0{\cdot}115b - 18{\cdot}5 \text{ in pounds and feet.}$$

This is, as indicated above, for single-seater aircraft. However, there is some indication that a successful man powered aircraft may have two seats. To what extent then is the above expression acceptable for two-seaters? I think it can be accepted that a wing of the same dimensions for a two-seater would weigh more because the payload carried would be approximately twice as much. The wing, however, would not weigh twice as much. The problem is how much more? Theoretical concepts are complex and only as reliable as the basic assumptions. I want to be as simple as possible in my approach, and therefore would like to fall back again on what actual facts are available. Morelli's work fortunately gives reliable expressions for empty weight for both single- and two-seater sailplanes.

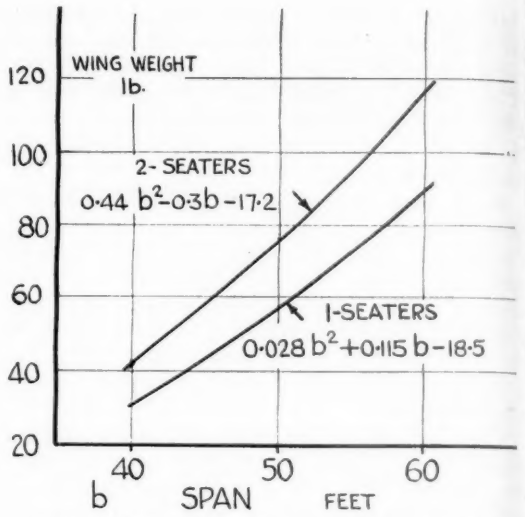


FIGURE 4. Wing weights for man powered aircraft.

It should, I suggest, be acceptably approximate if we examine how the ratio between these sailplane weights varies. An examination of the available data based on 24 single-seaters and ten two-seaters indicates that the empty weight of a two-seater is just about 1·5 times that of a single-seater. We may then take our original expression for a single-seat empty weight for man powered aircraft and multiply it by 1·5. The new expression would then be:

$$W_e = 5{\cdot}4b - 129 \text{ in pounds and feet.}$$

The ratio of wing weight to empty weight on Morelli's data on ten two-seaters is

$$W_w/W_e = 0{\cdot}133 + 0{\cdot}00815b$$

from which one can derive

$$W_w = 0{\cdot}044b^2 - 0{\cdot}305b - 17{\cdot}2.$$

We now have sufficient information to plot curves over the span range of from 40 to 60 ft. for man powered aircraft empty weights and wing weights for aspect ratios of the order of 15. These are shown on Fig. 4.

There being insufficient information available on the actual load factors used in the man powered aircraft built to date, it is considered to be unwise to try to vary these weights with differences in ultimate load factors. There is no doubt that the lower the load factor, the lighter the aircraft, but no information is available yet for load factors as low as 2·5 which have been suggested for man powered aircraft. It is probably safe enough to leave these expressions as correct, but in saying that, we must still admit that these weights shown in Fig. 4 are simply grasps at the unknown and are probably unreliable, but at least they are based on certain available data and are not entirely imaginary.

It should be mentioned here that to make this comparison between man powered aircraft and sailplanes valid, the man powered aircraft empty weights are based on only the following weights: wing, fuselage, tail, controls, skid undercarriage, ordinary pilot's seat, seat

belt and canopy, and do not include the following: wheeled undercarriage, drive to wheels, propeller, drive to propeller. For a two-seater, Nonweiler estimates these items to weigh about 30 lb.[5] The figure for the single-seat Haessler-Villinger was 24·2 lb. and for the single-seat two-propeller Bossi Bonomi, 35·8 lb.[6]

We are now in a position to see what we can do about balancing weight against drag and to see the order of weight that can be flown by one man or two men. For the sake of this discussion, I will assume only a cruising condition as determining the size and weight of the aircraft and not deal with the more difficult take-off condition in this paper. This problem deserves a paper to itself.

It may be assumed that, under cruising conditions, it is possible to take out the entire cruising power through the legs of the crew, which is a different condition from that occurring during the sprint power conditions at take-off. We shall assume that each man can produce in cruising 0·5 h.p.[5] These powers have been measured on such equipment as Prony brakes and therefore imply that these powers include certain frictional losses in the drive. However, in the drive to a propeller there might well be additional frictional losses, let us say 3 per cent. The propeller efficiency might be as high as 85 per cent so we can say that the power we hope will be used for the actual drive to the aircraft will be for a single-seater $0{\cdot}5 \times 0{\cdot}97 \times 0{\cdot}85 = 0{\cdot}41$, or for two men, 0·82 h.p. The aircraft will be required to use no more power than this when in level flight, but since it needs to cruise only at one speed, it is a reasonable first assumption that the above thrust h.p. would be made available at any speed in an analysis such as this. This is reasonable, because the range of speeds from which one could choose is very small indeed.

This gives us the basic facts or near-facts needed for design studies. I shall not, however, carry out a great deal of such work for the purpose of this paper because in spite of basic assumptions that may be agreed, there are many secondary and personal assumptions that have to be made. For instance, what degree of approximation is necessary for an acceptable answer?

To show the results of rough approximation, I have made one short study, using Nonweiler's drag data[5] and the weights that have been discussed. For a given span of 50 ft. and all-up-weight of 420 lb. for a two-seater I have allowed the aspect ratio to range between 5 and 20 which results in widely differing wing loadings. It will be clear that the much larger wing area for the low aspect ratio wing will weigh much more than smaller wings of the same span, so that in this respect I have favoured low aspect ratio. But I have given all aircraft the same parasite drag coefficient which favours the low wing area aircraft. This rough shot shows that the higher wing loadings require less power than the lower. This is not to be expected and may be wrong, but it does lead one to try a more precise method of comparison (Fig. 5).

For the sake of general interest, I have shown one of the machines as at 10 ft. altitude and also the power available from two men. Apparently a machine built to the assumptions I have made would fly for ten

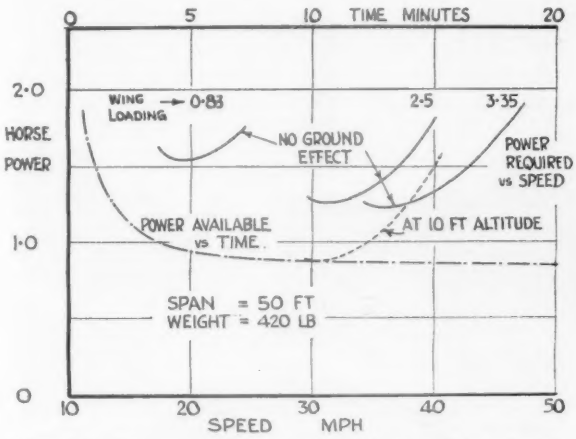


FIGURE 5. Performance curves estimated for a series of aspect ratios.

minutes and more at cruising man power with two men, using legs only.

If any general conclusion can be drawn, it is that the induced drag must be reduced as much as possible and that increase of span is the best way to do it even if the weight increases somewhat. It is, of course, the degree of "somewhat" which is unknown.

If span is to be kept as large as possible, the structural problem certainly becomes one of stiffness rather than strength and is therefore more complex.

4. Power Transmission

Pedals, chains and belts are the preferred methods for power transmission. This method is obvious to most people, except oarsmen who consider their motions more effective. Unfortunately, the rowing motion is not as effective as pedalling for driving a propeller[7]. Except for sprint conditions (up to a couple of minutes) there is no advantage in using more than your legs. Through the leg muscles, anyone can exert his full cruising power which is not limited by the number of muscles used but by the ability to use oxygen[8].

Examples of such transmissions which are worth studying are the Haessler-Villinger twisted belt scheme (Fig. 6) and the Bossi-Bonomi chain and sprocket scheme (Fig. 7). The chain shown leads through shafts and bevel gears to the propellers. The other sprocket was normally used to drive the undercarriage. Recently Perkins has used a plastic-covered rope as a belt. Nonweiler's scheme uses a series of chains and a final bevel gear (Fig. 8).

The problem of keeping the mechanism as compact as possible is so closely associated with aerodynamics that it cannot be dealt with by itself. On the other hand, I must try to deal with one thing at a time, so what I say about drives must be imagined within an aerodynamic scheme.

The final shaft is always above the heads of the crew, so there is always a gap of about 3 to 4 ft. between the crank shaft and the propeller shaft. The propeller tends to be well aft of the crew. The

FIGURE 6. Haessler-Villinger transmission.

simplicity of the Haessler-Villinger drive leads to inefficiency. What is really wanted is a twistable bicycle chain in light alloy. Otherwise the use of bevel gears seems almost inevitable. A chain with single instead of double links and overhung rollers is said to be easily twistable, but I have not seen one yet. The modern bead-link chain used for wash-basin plugs and also for plastic Poppets is twistable and may have development possibilities.

The actual efficiencies of various drives are most important. Seehase[9] has measured the mechanical efficiency of an initial chain drive plus two pairs of bevel gears as only 74 per cent. To improve this poor figure, he did two things: he used a link type instead of a roller chain and claims that that improved the chain drive efficiency from 92 to 99 per cent. He replaced his bevel gears with a theoretically impossible mechanism, two double-cranked shafts at right angles to each other, one driven by the other by connecting rods. Dynamically this is "No Go," for the connecting rods would have to change their lengths slightly during the stroke. He overcame this by incorporating rubber buffers in the rods, thus allowing the changes in length to take place. This scheme, he claims, gave an efficiency of 97·5 per cent, an overall drive efficiency of 96·5 per cent as against the original 74 per cent. Seehase also claims that the simple twisted belt drive, as can be shown by theory and test, is no more efficient than chains and bevels at the sort of belt speeds necessary in this sort of layout.

There does not appear to be a great deal of data available on mechanical efficiencies, but fairly recent work done at Oxford[10] on a 3-speed bicycle hub gear is worth recording here. The variation of mechanical efficiency with torque is shown in Fig. 9.

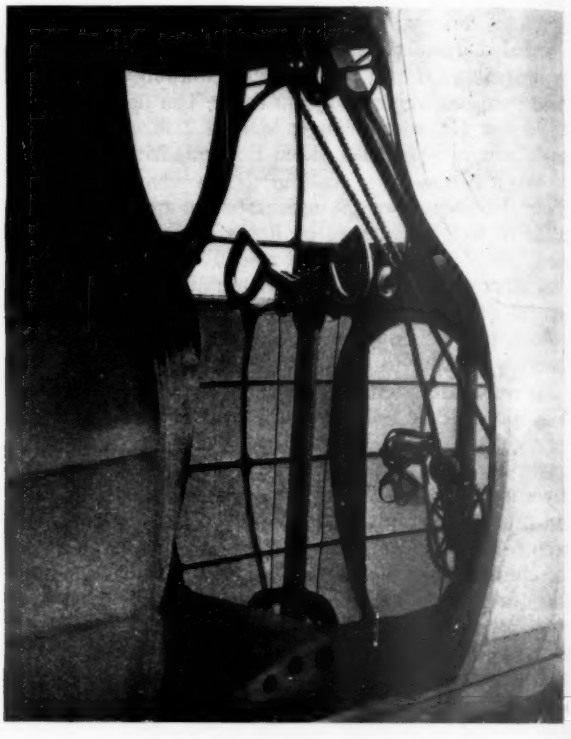


FIGURE 7. Bossi-Bonomi transmission.

The figures quoted in the foregoing paragraphs should not be accepted as gospel but are given to point out that any mechanism to be adopted should be bench-tested unless one does not care what comes out at the other end.

The propeller itself is a special problem. It should be as large as practicable, maybe up to 9 ft., and yet it has to deliver less than two h.p. It must be light and yet highly efficient, estimates showing that the efficiency should be somewhere in the 80s. If the drive is not very smooth the propeller will also have to act as a flywheel as it does for piston engines. Experiments and calculations have shown that the actual torque variations occurring with one pilot using arms and legs can cause a variation of two per cent in propeller efficiency through a revolution, although the net loss in

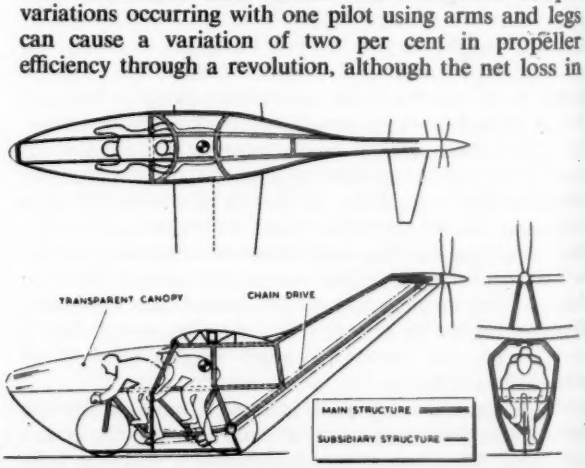


FIGURE 8. Nonweiler's transmission.

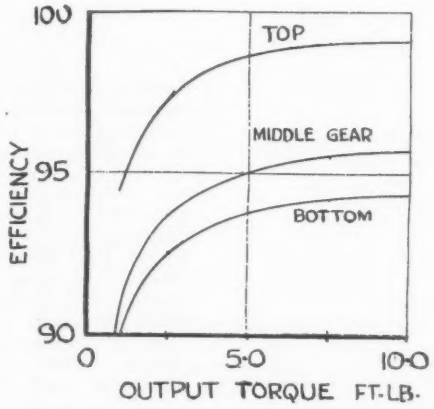


FIGURE 9. Efficiency of a 3-speed bicycle transmission.

this case was only 0·7 per cent. A larger diameter propeller with its weight concentrated near the tips would improve matters.

Conclusion

In devising a man powered aircraft, we have to struggle for every little gain that can be grasped. The structure must not cause aerodynamic losses, but the structure must be very light. The driving mechanisms must be light and unusually efficient and, in spite of a highly erratic torque, the propeller must not suffer. We have so little to play with, as in the years before 1910 when a flight was a flight and a great thing in itself. We have not even the consolation that our power plant will improve in foreseeable time. When success comes it will be only by good engineering, good planning and proper testing, rather than luck or the flash of inspiration.

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Man as an Aero Engine

D. R. WILKIE, M.R.C.P., M.B., B.S. (London), M.D. (Yale).

(*Department of Physiology, University College, London*)

Introduction

The first basis for the design of any power-driven mechanism is an exact knowledge of the characteristics and capabilities of the power unit available. Without such information no designer is able to start on the initial sketch of the mechanism. The man powered aircraft is no exception and its designers have suffered in the past by insufficient knowledge about man's power. This may seem remarkable because until about two centuries ago almost all the world's work was done by muscle-power, and much of the muscle was human.

This paper sets out to examine the properties of man considered purely as a source of mechanical power. Knowledge of these properties is of interest both in athletics and in the design of man-driven machines. The study was undertaken in order to find out theoretically whether it is possible for man to fly by his own efforts: probably it is[21, 24, 32].

[a]Based on a lecture given before the Man Powered Aircraft Group on 27th November, 1959. A similar paper based on the lecture is to be published in *Ergonomics*.

We all know that we can work harder for a short period of time than for a long one, but there are few systematic studies (*e.g.* Refs. 17, 20, 27, 28, 30) of the exact way in which power output diminishes as the duration of exercise increases. Extensive physiological studies have been made on running, but in this form of exercise little external work is done, so the results are of only indirect use for the present purpose.

Observations on various types of exercise, from many sources, are plotted in Fig. 2: the best performances at each duration have been extracted and plotted in Fig. 1. Note that, in both cases, the horizontal scale does *not* represent the time from the beginning of exercise, but the duration for which a given constant output can be maintained; a linear scale is used in Fig. 1 and a logarithmic one in Fig. 2. For a short period of time very heavy work can be performed, but the power output diminishes steeply the longer the total duration of exercise. However, when the duration of working is greater than about 5 min., the rate of working diminishes only very slowly with increasing duration of exercise. In order to understand these observations, it is necessary

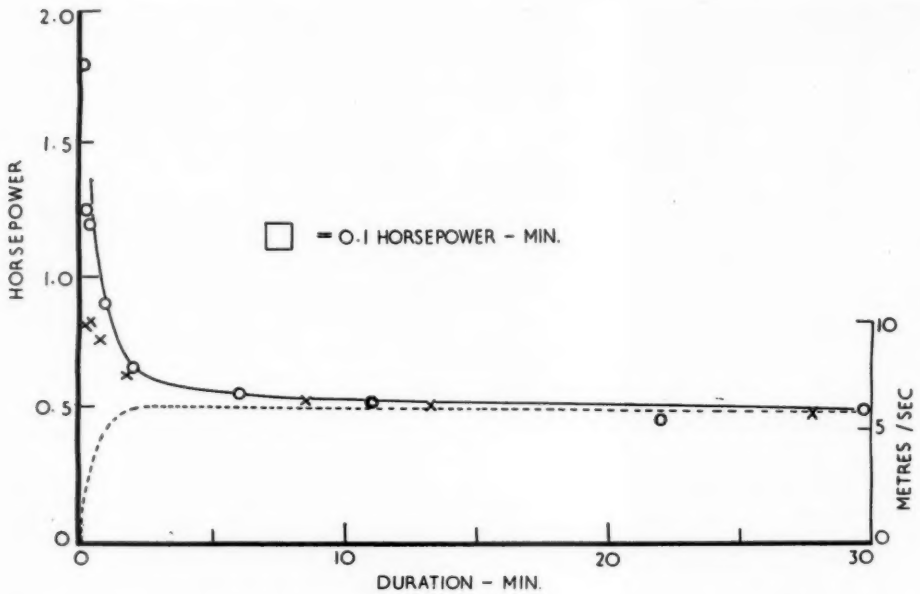

FIGURE 1. Left-hand ordinate; circles, maximal external mechanical power produced by champion athletes, data extracted from Fig. 2. Right-hand ordinate; crosses, running speed, world records[12]. Abscissa; total duration of exercise (*not* time elapsed since the beginning of exercise). The broken line shows the energy available from oxidative processes. To this is added 0·58 h.p. minutes of work from anaerobic (hydrolytic) sources, to give the theoretical curve, full line.
N.B. 1 h.p. = 0·746 kilowatts = 76 Kg.-wt. metre, sec.$^{-1}$.

to know something of the physiology of muscular exercise.

The Physiology of Muscular Exercise

The function of muscles is to transform chemical into mechanical energy. All the chemical processes take place at constant temperature, that is, their energy does not appear at an intermediate stage as heat. For this reason muscle is quite unlike a heat engine. Though the chemical processes involved are very complicated, muscle achieves an efficiency (=work output/chemical energy used) of 20 to 25 per cent under favourable conditions: the chemical energy comes ultimately from the oxidation of foodstuffs, most probably of the carbohydrate glycogen and fats, to carbon dioxide and water.

From the long-term point of view, energy production by muscle thus depends on an adequate supply of oxygen, which must be absorbed at the lungs and transported by the blood-stream to the active muscles. Both lungs and blood-stream have a limited capacity, which in turn sets a limit to the steady-state energy production. Fit young men can absorb up to 4 litres of oxygen per minute[3,25], the maximum absorption that has been recorded is 5·4 litres/min., by an Olympic athlete. Since 1 litre of oxygen yields about 0·1 h.p. minutes of mechanical work under optimal conditions, the steady-state power output must be limited to 0·4 – 0·54 horse power, depending on whether we are considering fit ordinary men or champion athletes. This prediction correlates well with direct measurements of mechanical power output (*see* Figs. 1 and 2).

Chemical Considerations

Although the ultimate source of muscular energy is oxidation, the immediate source of energy is the hydrolysis of various compounds, such as adenosine triphosphate, creatine phosphate and, when oxygen is lacking, most important of all, the hydrolysis of glycogen to lactic acid (*see e.g.* Ref. 18). The *rate* of these hydrolytic reactions is not limited by the supply of reactants from outside the muscle, and it may therefore be very high, though the *total amount* of energy available is limited by the amounts of such chemicals stored in the muscle. As shown in Fig. 1 the experimental measurements of maximal work-production (circles) satisfactorily fit a theoretical curve (full line) constructed on the assumptions:— (*i*) that there is a steady oxidative energy production of 0·5 h.p. falling to 0·475 h.p. after 25 min. as a result of long-term fatigue (broken line); (*ii*) that to this is added a "lump-sum" of mechanical work derived from hydrolytic reactions. This amounts in practice to about 0·6 h.p. minutes, which can be released over a long or short period, according to need.

The theoretical calculation is made slightly more complex by the fact that the oxygen consumption provoked by exercise does not rise instantly to its full value as soon as exercise begins. It rises instead with a roughly exponential time-course (half-time approximately 30–40 sec.)[14]. Thus during a short bout of maximal exercise a disproportionate amount of energy has to come from anaerobic processes[23]. Since the resulting metabolites are initially responsible for stimulating the increase in oxygen intake, this increase does not occur until the metabolites have accumulated to some degree.

The experimental points in Figs. 1 and 2 both refer to a steady rate of power production, maintained for the duration indicated. A theoretical analysis into oxidative and hydrolytic components makes it possible to forecast the limits of performance if the task set involves non-uniform power production; in this case, the hydrolytic "lump-sum" must be distributed in an appropriate way as a function of time.

Of course, the stores of hydrolysable chemicals must be replenished after the exercise is over, the energy

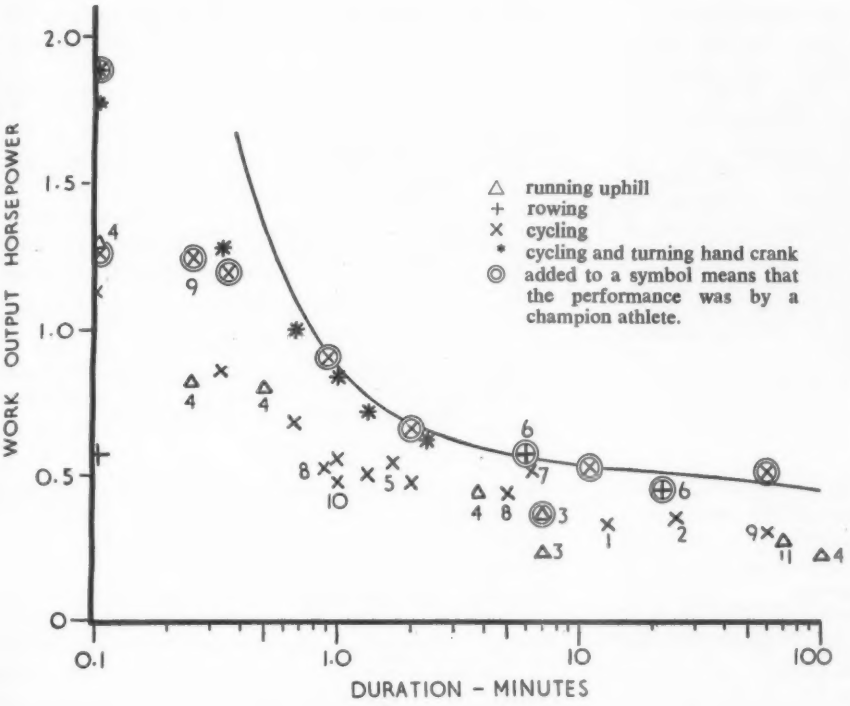

FIGURE 2. Maximal output of external mechanical power (h.p. linear scale) plotted against total duration of exercise (min. logarithmic scale). The logarithmic scale has been used to display the experimental points clearly. Full line is the theoretical curve shown in Fig. 1. The numeral indicates the source from which the experimental point was derived. Points without numerals: champion cyclists, ⊗ Ref. 21; ordinary cyclists, ×, also ordinary cyclist performing hand-cranking in addition, * Ref. 29.
1, Abbott *et al*; 2, Asmussen; 3, Bannister; 4, Benedict *et al*; 5, Bonjor; 6, Henderson *et al*; 7, Karpovich *et al*; 8, Nielsen *et al*; 9, Raleigh; 10, Tuttle; 11, Unna.

needed being obtained from additional oxidation. Thus the "oxygen debt" accumulated during exercise is paid off during recovery. The maximum oxygen debt that can be accumulated is about 20 litres. This might be expected to yield about 2 h.p. minutes, but we have seen that in practice only a third of this (0·6 h.p. minutes) is actually obtained as external mechanical work. The reason for the inefficient utilisation of the oxygen debt is not altogether clear.

Mechanical Considerations

The speed with which a muscle shortens depends on the force to be overcome: the larger the force, the slower the shortening, and vice versa. No work is done if the force is zero, or if the force is so great that the speed is zero; though chemical energy will be consumed almost as usual[31]. In order to achieve an optimal conversion efficiency of 20 to 25 per cent, force and speed of movement must be suitably matched to one another. It so happens that the optimum occurs when the force has about one-half, and the speed of movement has about one-quarter, of their respective maximum values[15]. For the greatest power output, regardless of economy, the force should be somewhat less and the speed somewhat greater. When using a machine, such as a bicycle, it is usually possible to adjust the gearing so that the load is matched to the muscles; but in many athletic pursuits this is not the case. Some examples will be discussed later.

It should also be noted that the conversion from chemical to mechanical energy in muscle is a one-way process. Animals are obliged to employ reciprocating movements, not rotations, so the kinetic energy of their limbs is continually altering. Energy given to accelerate a limb will inevitably be wasted unless there is a mechanism to decelerate the limb again and store the energy. If the movement has to be checked by the contraction of antagonistic muscles, these will use up yet more energy, not gain it[1].

Experimental Data

In presenting the data collected from many sources an attempt has been made to distinguish between the results obtained from different types of exercise, and also to distinguish champion athletes from healthy non-athletes. This last distinction may seem somewhat arbitrary, but when it is made, the experimental points fall into two fairly well-defined sets. The champion athletes (double circles in Fig. 2) can develop 20 to 30 per cent more power than healthy normal men can, for a given type of exercise.

RUNNING (Fig. 2; triangles)

Running is not a good way of producing external mechanical work. Sprinting at 7 m./sec.—a speed which can be kept up for perhaps 30 sec.—only 0·16 h.p. appears as external work, in overcoming air resistance. At the same time an estimated 2·4 h.p. is dissipated internally; 0·6–0·7 h.p. in raising and lowering the centre of gravity, and 1·7 h.p. in changing the kinetic energy of the limbs[9]. Sprinting against an artificial resistance has yielded 0·31 h.p. of external power[7], but this must also be only a small fraction of the actual mechanical power developed. However, the metabolic changes resulting from running have been extensively studied (*e.g.* Refs. 4, 14, 23). Fast running has been shown to be very uneconomical; a large increase in energy expended leads to only a small increase in speed.

The same effect can be seen in Fig. 1 (crosses). At speeds less than about 6 m./sec., running speed decreases in exactly the same way with increasing duration of exercise as does the working ability of the body. But in short bouts of exercise (duration less than 1 min.), running speed falls off compared with the ability to perform external work (compare crosses and circles in Fig. 1). The chief importance of the crosses in Fig. 1, is that they support the conclusion that a rate of working that can be kept up for 10 mins. can almost be kept up for 30 mins. or even an hour.

The triangles in Fig 2 all represent work done while running uphill. Normally this activity merely leads to an increase in the potential energy of the body, but it could, in theory at least, be made available by using a treadmill.

ROWING

Rowing, using a sliding seat (Fig. 2; upright crosses) is an effective method of producing external mechanical work so long as the duration is more than two or three minutes. For shorter bouts it is very uneconomical because of the disproportionate wastage of energy from acceleration and deceleration of the whole body that results when this type of movement is made at high frequency[28]. The advantage that might have been expected theoretically from the use of a larger muscle-mass is thus not obtained in practice.

PEDAL-CYCLING (Fig. 2; diagonal crosses)

Cycling can be adapted to various durations of maximal effort. The mechanical system involved is simple and lends itself to the free use of the power obtained; gearing can be readily adjusted; maximum use can be made of the kinetic energy of the moving limbs. In cycling the full steady-state power production of the body can be developed. Thus the muscle-mass of the legs is evidently more than large enough to utilise all the oxygen that can be absorbed. The maximum rate of pedalling, under no-load conditions, *e.g.* in roller-racing, is about 180 r.p.m., the optimum for greatest efficiency is about 60 r.p.m., and the speed used in cycling contests (where economy may have to be sacrificed to output) varies from 60–120 r.p.m.

PEDAL-CYCLING WITH HAND-CRANKING (Fig. 2; stars)

The total amount of energy immediately available from hydrolytic reactions is limited by the initial size of the chemical stores in the *active* muscles. Therefore, maximal use of hydrolytic energy sources can be made only by employing and exhausting as large a mass of muscle as possible. So for short bouts of exercise, the more muscle employed, the greater the power developed. Accordingly, it has been found[28,29] that simultaneous cycling and hand-cranking yields about 50 per cent more power than cycling alone, but only for a short time; Ursinus[28,29] found that the advantage is rather small after 5 to 10 min., when the power output becomes limited by oxygen supply rather than by muscle-mass. Four different types of machine for combined arm and leg movement were thoroughly investigated by Ursinus, who concluded that simple rotation of a 17·5 cm. crank by hand and foot was the most effective arrangement, as well as being mechanically the simplest (Fig. 3). He also established the optimal speed for each duration of effort; the best phase relation between arm and leg; and the scope and limitations of various postures, ranging from lying on the back, through normal sitting, to lying face downwards. Most of Ursinus' experiments were made on a subject who, though not an athlete, was yet able to produce for two minutes a power output equal to that of the best athletic cyclists using the legs only. If cycling athletes were to benefit to a similar degree from the use of their arms as well as their legs, this would substantially raise the experimental points at the extreme left-hand side of Figs. 1 and 2, so that they fell closer to the theoretical curve.

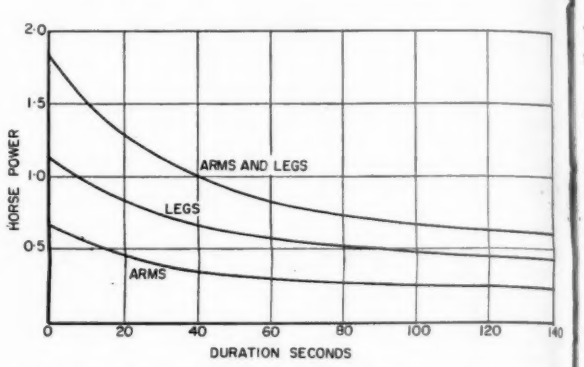

FIGURE 3. Comparison of horse power produced from work done by legs alone, arms alone and arms and legs combined. Redrawn from Ursinus. Ordinary subject.

Efforts of Short Duration

The points plotted at 0·1 min. in Fig. 2 are not strictly comparable with one another. They represent maximum peak outputs whose short but uncertain durations depended on many factors, for example, on the inertia of the apparatus. The bout of exercise will in each case have consisted of a number of repeated movements. It is of interest to consider what may be the power output of the body during a *single* movement, occupying a second or less. In this case the power is limited only by the mass of muscle that can be brought to bear effectively, for fatigue can hardly arise within such a short time. A theoretical upper limit of about 11 h.p. is set by two facts: that the body is about 45 per cent muscle; and that human muscle can develop about 0·3 h.p. per kilogram when it is shortening against a matched load[31]. The full amount of power could only be extracted, of course, if it were possible to connect every muscle in the body to a suitable load, then to throw them all into contraction at once. The mere fact that muscles are almost all arranged in the body in antagonistic pairs may be expected to reduce the theoretical limit for any practical movement to perhaps 5 or 6 h.p.

In order to investigate the peak power output of the body in more detail, it would be necessary to construct a machine for the specific purpose of absorbing the

work from arms, torso and legs at optimal speed; it seems that no existing athletic task is suitable.

Conclusion

It is deduced from the published literature that the usable external power output of the body is limited in the following manner for the reasons stated :

(1) In single movements (duration less than 1 sec.) to less than 6 h.p.; by the intrinsic power production of muscle, and by the difficulty of coupling a large mass of muscle to a suitably matched load.

(2) In brief bouts of exercise (0·1 – 5 min.) to 2 – 0·5 h.p.; by the availability in the muscles of stores of chemical substances that can yield energy by hydrolysis.

(3) In steady-state work (5 min. to 150 min. or more) to 0·5 – 0·4 h.p.; by the ability of the body to absorb and transport oxygen.

(4) In long-term work, lasting all day, to perhaps 0·2 h.p.; by wear and tear of muscles, the need to eat and so on.

All these figures refer to champion athletes; ordinary healthy individuals can produce less than 70 – 80 per cent as much power.

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Some Problems of Polar Missile Control*

by

D. BEST, B.Eng.

(Chief Engineer, Ferranti Ltd.)

1. Introduction

A polar-controlled missile is one in which manoeuvre is carried out by rotations about roll and pitch axes, that is, in the manner of a conventional aeroplane. This paper discusses some problems in the application of this form of control to homing missiles.

In comparison with the alternative Cartesian configuration, this method presents some special design problems. In the former case, it is often possible to resolve the motion into two planes and consider the pitch and yaw control systems as independent two-dimensional problems. This simplification is not possible in the case of polar control and it is usually necessary to consider the whole three-dimensional system. The equations of motion which result are, in general, not susceptible to analysis. Because of this, the design of control systems requires extensive use of simulators.

2. Graphical Representation

In the discussion which follows, diagrams are used to represent three-dimensional relationships. Two methods of projection are in common use. The first is used where the quantities of interest are angles and rotations, ignoring distances. For example, in a homing trajectory, the principal directions of interest are the missile and target directions of motion, and the line of sight between missile and target. These are regarded as points on the surface of a sphere projected on to a plane (Fig. 1). Thus, points on the diagram represent directions in space, and lines represent rotations. It is convenient when plotting trajectories to take the line of sight as the centre of the projection since it is then possible to draw a perfect proportional navigation homing course as a straight line.

Thus, referring to Fig. 1, if the point *S*, the centre of the projection, represents the line of sight between missile and target, then *M* and *T* may represent missile and target directions of motion respectively. It will be seen that *ST* represents, to a suitable scale, the component of target velocity normal to *S*, so that for the missile to achieve interception it must change its direction of motion such that the component of missile velocity normal to *S* is ST_1 to the same scale. Therefore, for equal missile and target velocities, the missile trajectory required for interception is *MT*. Most of the diagrams depicting angular relations used here are constructed in this way, using either the line of sight or the roll axis of the missile as the centre of projection.

The second method (Fig. 2) projects missile and target directions on to a plane containing the line of closest approach assuming a straight course. This is an approximate representation which assumes deviations from initial conditions to be small, but it is convenient for assessing miss distance. This is usually known as a " bore sight " diagram.

To illustrate the relation between these two diagrams, Fig. 3 shows the same trajectory plotted by both methods. In both cases, missile roll attitude is indicated

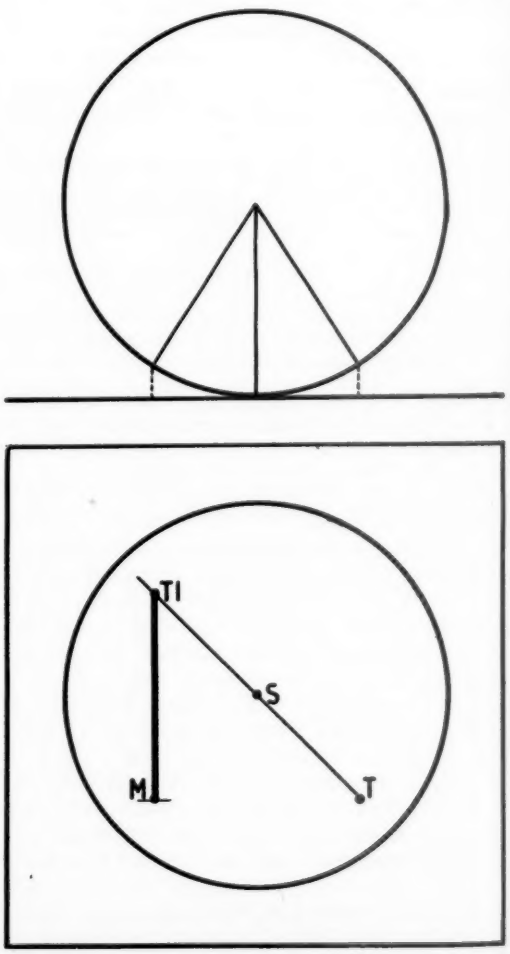

FIGURE 1. Angular projection.

*The 11th lecture to be given before the Astronautics and Guided Flight Section on 18th February 1960.

by a representation of the wing plane. In the case of the angular projection, it is convenient to use a scale of velocity rather than angle.

Trajectories plotted on these diagrams are useful both for preliminary assessment of control methods and for examination of trials results.

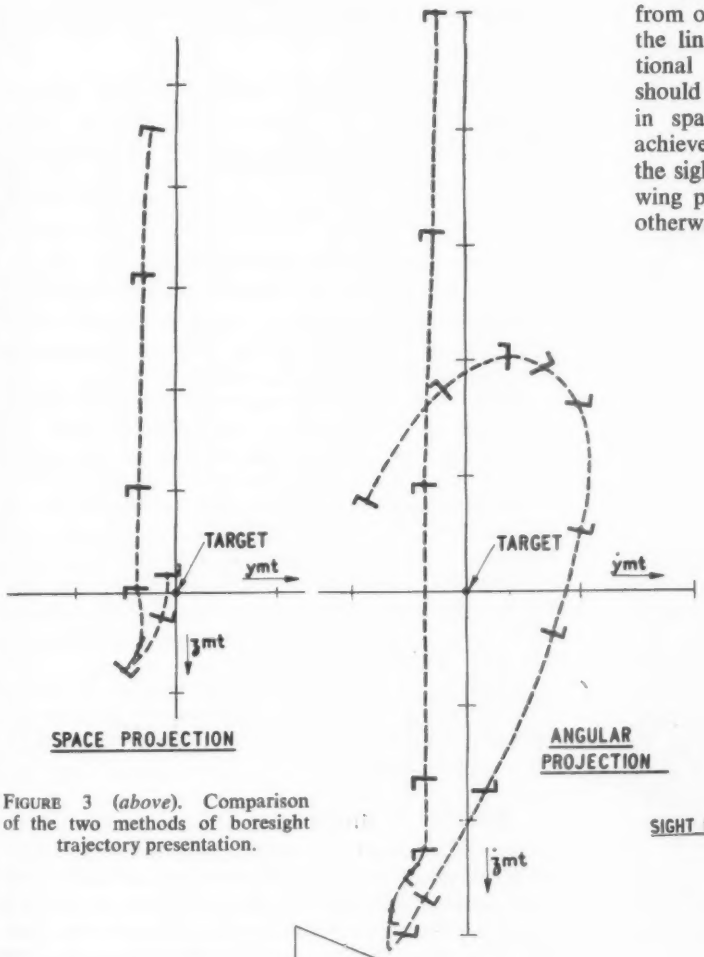

FIGURE 3 (*above*). Comparison of the two methods of boresight trajectory presentation.

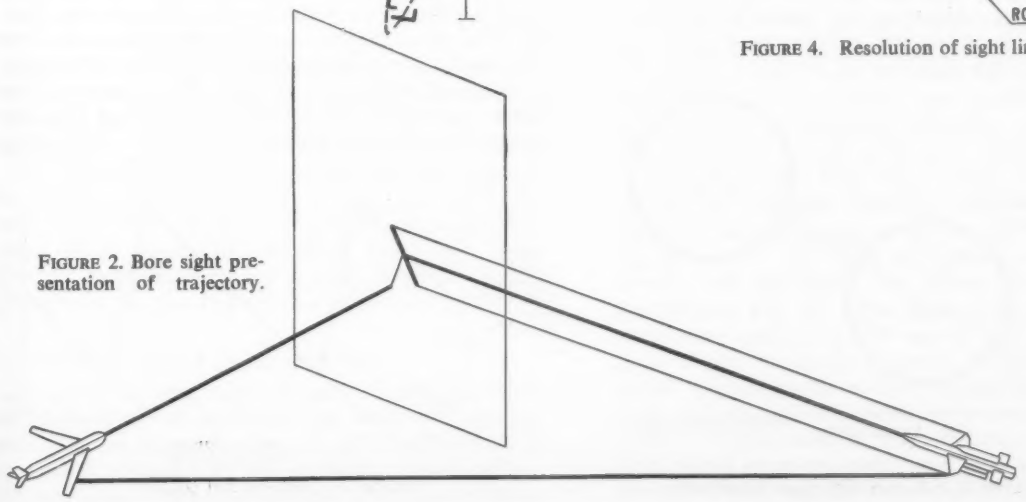

FIGURE 2. Bore sight presentation of trajectory.

3. Roll Control Methods

In a polar control missile the accuracy and speed of roll control has a most important effect on performance, and it is this aspect which forms the main topic. We first consider the way in which a roll control function is derived from the guidance information.

A homing missile derives its guidance information from observation of the rate of change in direction of the line of sight from missile to target. The proportional navigation method requires that the missile should turn at a proportional rate about the same axis in space. In a monoplane missile, this must be achieved by rotations on roll and pitch axes. Thus, the sight line spin vector must always lie in the missile wing plane if the missile is to perform correctly, since otherwise yaw manoeuvre would be required to satisfy

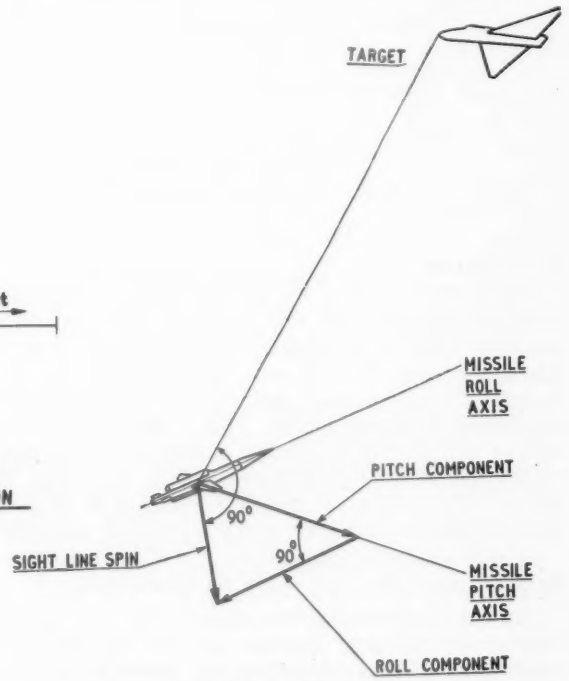

FIGURE 4. Resolution of sight line spin.

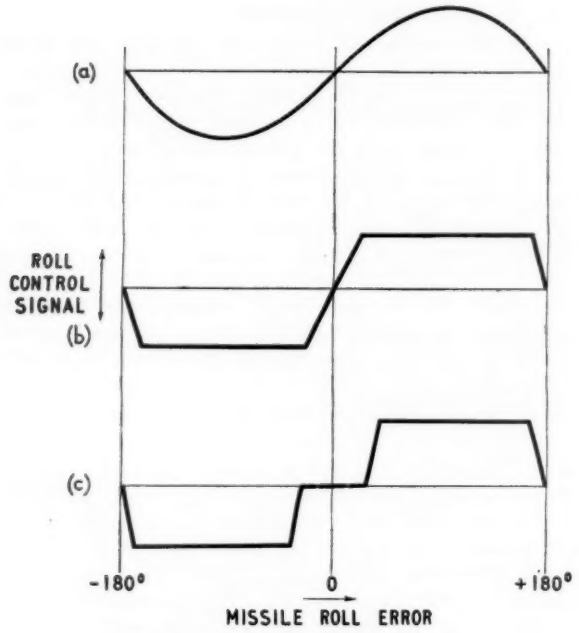

FIGURE 5. Roll control functions.

the navigation law. The correct orientation of the missile is shown in Fig. 4. It is the function of the roll control system to satisfy this condition. The missile is therefore made to roll until the yaw component of sight line spin is zero. It is obvious that if the sight line has no spin, there is no yaw component at any roll angle and therefore no roll control. Thus, if the yaw component is used directly as the roll control signal, the gain of the roll servo is proportioned to the magnitude of the sight line rate, and a system which is stable at high rates will have a very poor response for lower demands.

Various ways of minimising this effect have been proposed and some of these are illustrated in Fig. 5.

(*a*) shows a sine function of roll error derived by varying the roll gain inversely as the magnitude of total spin. This is satisfactory for small errors, but large corrections from near 180° tend to be slow.

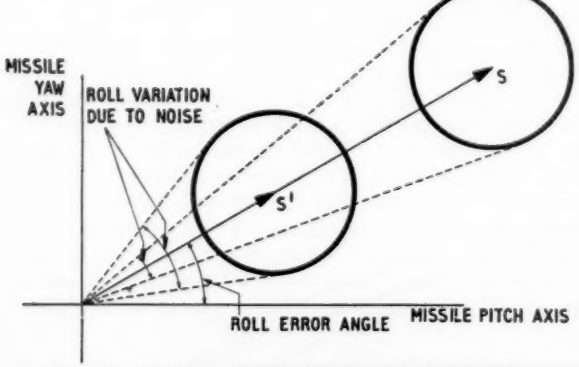

FIGURE 6. Variation of roll noise with signal amplitude.

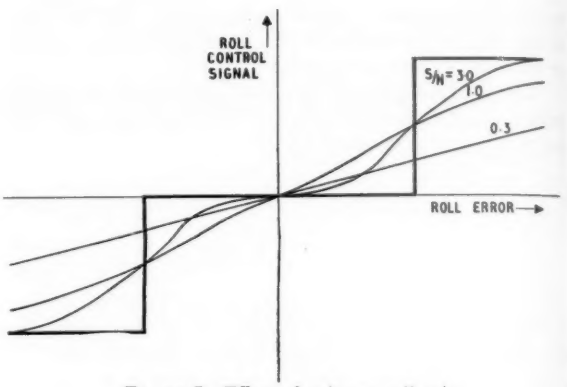

FIGURE 7. Effect of noise on roll gain.

(*b*) is achieved by limiting and high gain. Oscillation can still occur at high rates, but the amplitude is stable because of saturation. The provision of a small dead space, as in (*c*), reduces the oscillation to a smaller amplitude at the expense of a small zero error.

There is no end to the variety of control functions which can be devised and it is not easy to decide between them. In practice, the differences are not as great as might be supposed because they are masked by the random variation in signal, which is often large. In any system which measures a function of roll angle, this variation may be greatly magnified at small rates. If the circle in Fig. 6 represents the range of random variation of mean sight line spin vector *S*, it will be seen that at low rates the variation in demanded roll angle can be very large indeed.

The effect of noise on a control function of type *C* is shown in Fig. 7. As the noise content increases the average control function becomes more linear, and the effective gain is reduced.

4. Bistable Control

The roll control functions discussed so far allow only one stable position in roll. If the missile is symmetrical so that it can pitch equally well up or down, a control function having two stable states 180° apart is possible (Fig. 8). In this case the maximum roll error is 90° and faster roll response is possible. This does not necessarily improve the overall response of the missile. Consider, for example, the effect of a sudden reversal of manoeuvre demand on a missile with moving

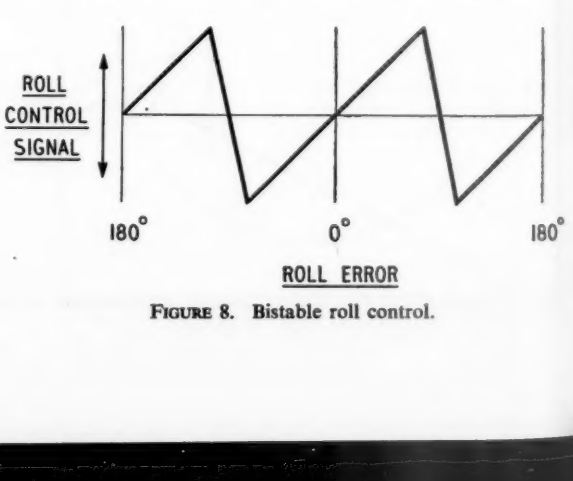

FIGURE 8. Bistable roll control.

wings. The monostable missile achieves this by rolling through 180° with no change in demanded pitch wing angle. In the bistable case, no roll is necessary, but the pitch wing angle must reverse. Which represents the faster response depends on the relative speeds in roll and pitch. In homing missiles, especially at high altitudes, response in roll is commonly faster than that in pitch. In these circumstances, the monostable system may well give better performance.

5. The Effect of Roll Errors

We have seen that a variety of roll characteristics are possible, and we now consider the roll performance necessary to achieve accurate interceptions. The important parameters are

(*a*) static errors in roll,

(*b*) response to transient disturbances,

(*c*) maximum roll rate.

In a perfect missile, once the initial misalignments are corrected, the rate of roll demanded to achieve a good interception never exceeds a few degrees per second. Unfortunately, the shortcomings of a practical design considerably alter the picture. Consider, for example, the effect of errors in roll angle which may arise because of drifts or biases in the airframe roll control system. Errors in the measurement of sight line rate causing yaw bias may also cause roll errors. These effects result in the missile taking a spiral course towards the target (Fig. 9) with continuous roll motion. The degree of error which can be tolerated depends on the particular characteristics of the missile system. In general, the rate of roll increases as the missile approaches the target. In a low gain roll system this may result in such an increase in roll error that the missile assumes a helical course, resulting in a large miss. In most cases, however, roll errors less than about 20° have no appreciable effect on accuracy provided good response can be maintained.

Thus, the need for small dynamic errors is governed to a large extent by the probability of static errors. To take another example, it can be shown that time lags in the pitch control system lead to reversals of manoeuvre demand in the end course phase immediately before interception. In order to satisfy these demands, the monostable missile must roll rapidly through 180°. This calls for large control signals at maximum roll error.

To summarise, the requirements for roll response and maximum rate of roll are set by system errors and biases. A perfect missile on a proportional navigation course would require to roll at no more than about 10°/sec. In practice, rolling at many times this rate is necessary in the last few seconds before impact.

6. Homing Head Requirements

An essential feature of a homing missile is a stabilised platform which maintains the target seeker accurately pointing along the line of sight independent of the missile motion. If this isolation is not perfect, couplings arise which may greatly affect performance.

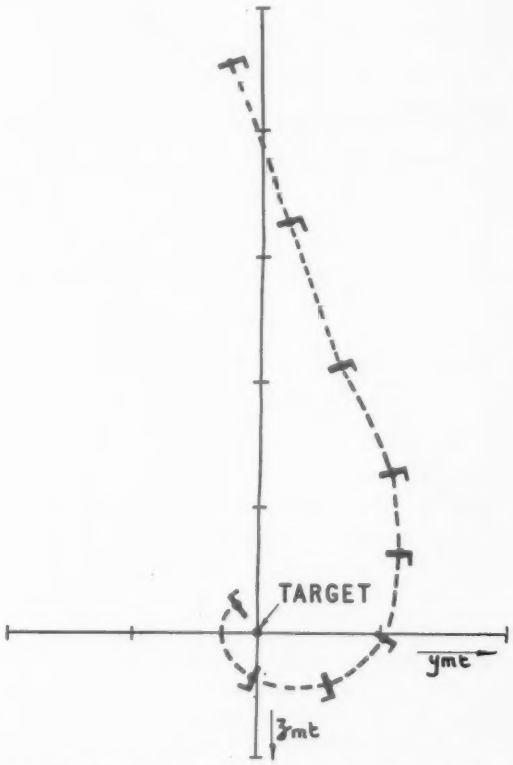

FIGURE 9. Effect of roll error on the space projection boresight plot.

The high rate of roll required for a polar-controlled missile imposes special restrictions on the design of the platform. A normal two-axis mounting with axes in the pitch and yaw planes requires large accelerations to stabilise against high roll rates when the sight line is displaced from the missile axis. This is illustrated in Fig. 10, which shows the angular velocities and accelerations in the gymbals of a two-axis mounting in a missile rolling at constant rate $\dot{\phi}$. The stabilising power required is approximately proportional to the velocity-acceleration product, and it will be seen that the power required increases rapidly for angles of displacement above about 50°, and this sets a practical limitation on the maximum squint angle. A three-axis mounting would avoid this difficulty, but at considerable expense in complication and weight.

It will be noted from Fig. 10 that as a consequence of the high angular velocities and accelerations, the errors in the stabilising servos caused by missile rolling increase as the sight line displacement from the missile axis increases. The effect of servo error is illustrated in Fig. 11. This figure represents an angular projection centred on the missile roll axis *O*. The point *T* represents the direction of sight line to the target. Under steady conditions the target seeker axis *D* is colinear with *T*. Servo error due to missile roll causes a displacement of *D* which appears in the homing head output as an apparent target movement. This can produce a false demand for roll in the direction shown

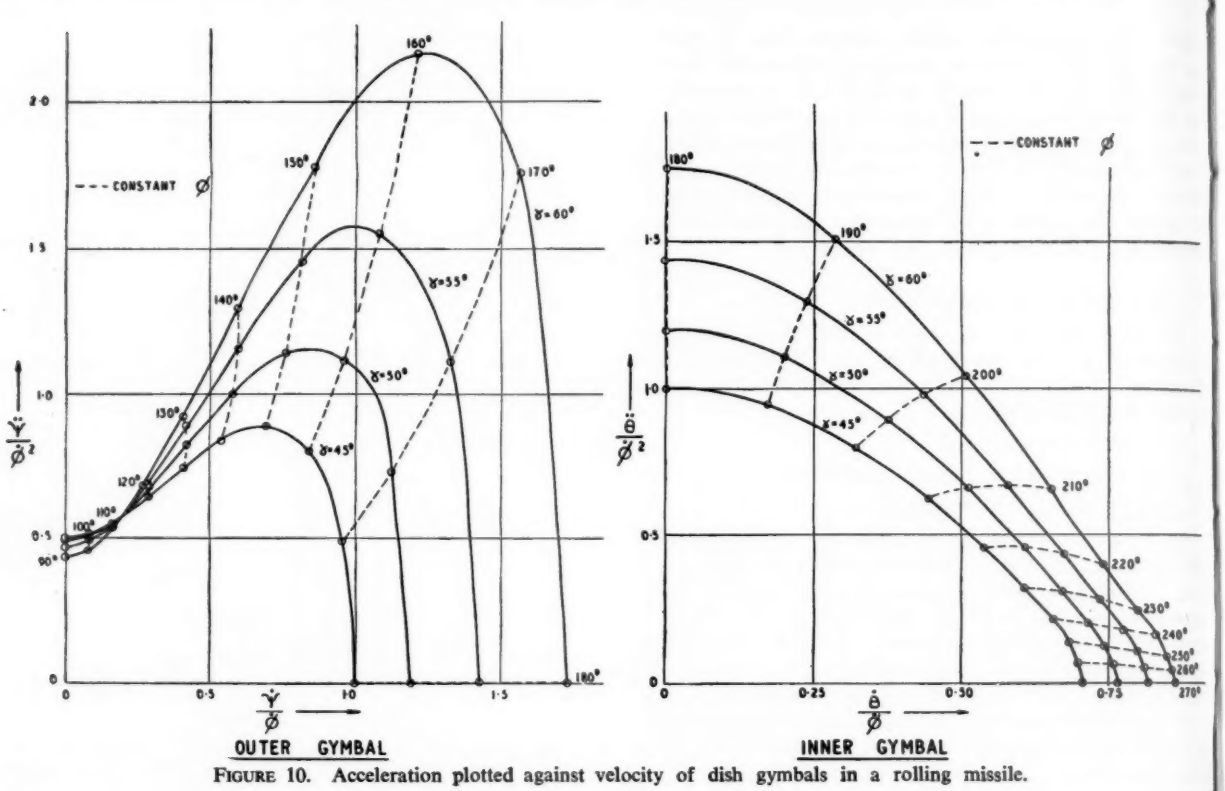

FIGURE 10. Acceleration plotted against velocity of dish gymbals in a rolling missile.

on the diagram, which may cause instability. It will be seen from the figure that the sense of the feedback depends on the directions of displacement of the sight line from the wing plane. Both senses of feedback may cause oscillation in roll.

The destabilising effect of this coupling is therefore dependent on both the displacement of the sight line from the missile axis and the relative roll attitude. A typical stability diagram is shown in Fig. 12. This indicates boundaries of stability in relation to displacement of sight line from the missile axis.

7. Collimation Errors

Another type of roll coupling is introduced by errors in alignment between the stabilising gyro and the target tracking head. As the missile rolls, the gyro axis remains fixed in space, but unless the target seeker

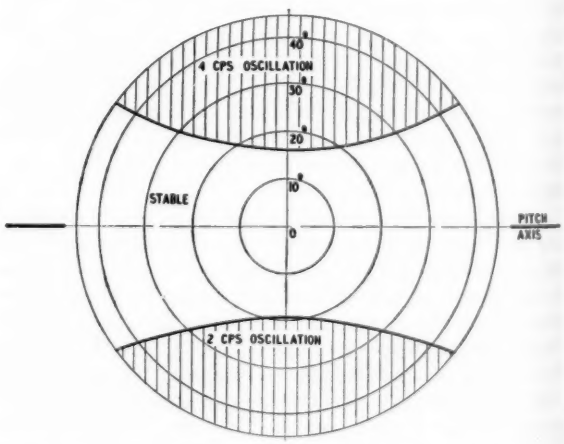

FIGURE 12 (*above*). Servo coupling stability boundaries.

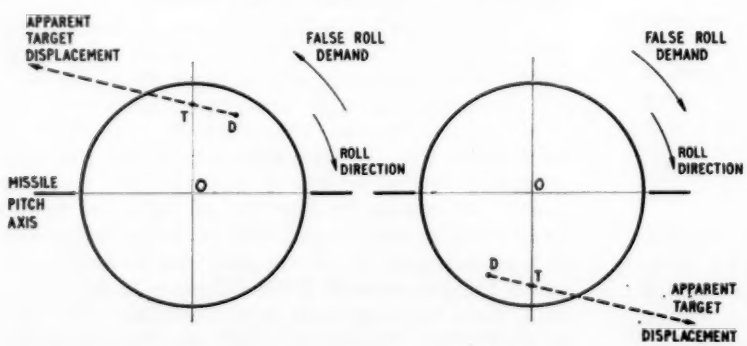

FIGURE 11 (*left*). Homing head roll coupling.

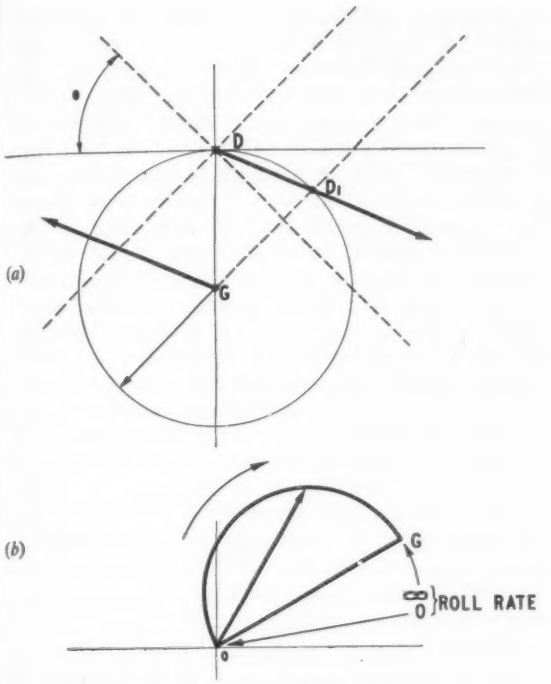

FIGURE 13. Collimation error.

axis is exactly parallel it will tend to be displaced, causing an apparent target manoeuvre. This effect is illustrated in Fig. 13. Referring to diagram (*a*), the sight line direction is shown along the missile axis and, initially, the tracker axis *D* is correctly aligned in the same direction. Due to collimation error, however, the gyro axis *G* is displaced through an angle *DG*. If now the missile rolls suddenly through an angle θ, the gyro axis remains fixed in space, but the seeker axis is displaced to D_1. This results in an apparent target displacement causing the gyro to precess in the direction indicated by the arrow. The result is a spurious demand which, at a steady roll rate, is in a fixed direction relative to missile axes. Both magnitude and direction, however, vary with rate of roll. Fig. 13(*b*) shows in missile axes the locus of the spurious demand vector with varying rate of roll, where *OG* is the direction of collimation error.

The effect of this type of error is a tendency either to continuous rolling or to sluggish roll response, depending on the direction of the misalignment. The behaviour of the missile is modified by the presence of a true sight line rate. In Fig. 14, with no true signal, the collimation offset *OG* produces a positive feedback, causing continuous rolling. A small signal *A* prevents rolling unless the roll rate initially exceeds a threshold, since rolling ceases if the total demand vector crosses the pitch axis. This occurs if the rate of roll is less than r_2. A large signal *B* prevents rolling under all conditions since, even at infinite roll rate, the total signal still crosses the pitch axis.

These effects become serious unless collimation accuracy is held to a few minutes of arc.

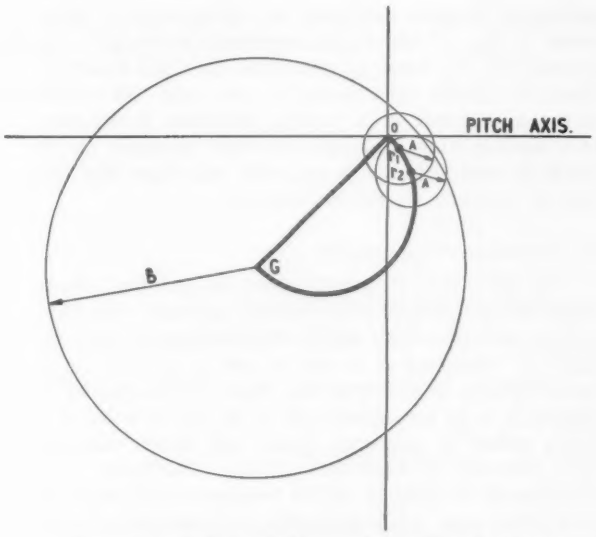

FIGURE 14. Roll threshold caused by collimation error.

8. Aberration

In a homing missile, the target is observed through a transparent nose which produces an apparent offset in the target direction. The amount of offset is usually not constant at all angles of look, so that angular motion of the missile body causes spurious target movements to appear.

The effect of this aberration on the normal acceleration control loop is well known. In a polar control missile, however, any motion of the body in yaw causes a false yaw component of sight line rate which in turn causes the missile to roll. Unfortunately, it is all too easy to induce yawing motion by a combination of pitching and rolling, so that the possibility of an unstable feedback loop exists (Fig. 15). In a fixed-wing missile, yawing is inevitable when a roll manoeuvre is

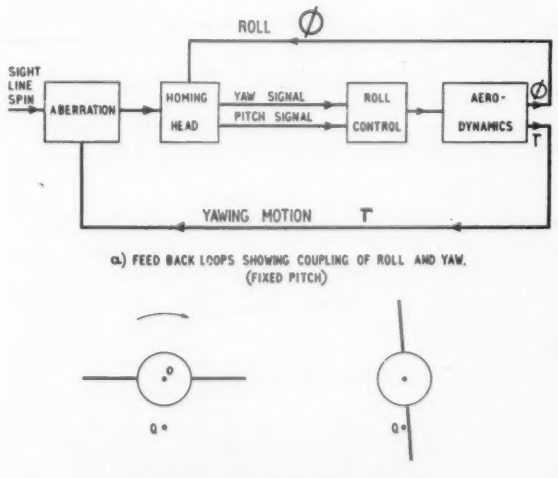

FIGURE 15. Yaw coupling.

performed at pitch incidence, an effect which is illustrated in Fig. 15 (*b*), which represents a missile flying at incidence *DQ* where *Q* represents the flight direction. Since the missile rolls about its own axis, this results in the appearance of a yawing incidence component. In a moving wing configuration body incidence can in theory be zero, but this is not easy to achieve and this type of coupling cannot be neglected.

9. System Assessment

We have seen three examples of coupling effects which are peculiar to polar control systems. All have inherent non-linearities which make assessment specially difficult. Although it is easy to see in general terms that couplings of this type can cause undesirable oscillations, it is by no means easy to decide how far it is worth going to suppress them. All these examples show threshold or limit cycle effects. Oscillations may be initiated or damped out by random disturbances, or may occur only at a particular combination of sight line angle and missile attitude.

As an illustration, consider the effect of collimation error. To avoid altogether the possibility of continuous rolling, demands extreme accuracy of alignment, which is difficult to achieve. Nevertheless, the presence of a sufficiently large demand will cause the missile to align itself correctly. It is, therefore, self-correcting in that divergence from course as a result of rolling ultimately causes a demand large enough to stabilise the missile. Whether or not this is tolerable depends on the particular trajectory of the missile, the sense of collimation error and on whether random roll motion causes the threshold to be exceeded. The problem is to set limits on collimation error which result in acceptable performance under probable interception conditions.

The use of simulators can help to assess this kind of problem, but can never hope to give a clear answer. Indeed, unless great care is taken, simulator work can be positively misleading. To make a quantitative assessment, it is necessary to have a very detailed model of missile behaviour, which accurately reproduces limits, biases and random variations which exist in practice. Such a model has so many degrees of freedom that some limitation must be put on the range of investigation. It is all too easy to accumulate a mass of useless results if the work is not carefully planned. This planning is greatly aided by preliminary work on a simple simulator in which much of the detail is omitted. A machine of this type will not generally produce useful quantitative results, but only serves to establish trends. A programme of detailed investigation can be drawn up on this basis: it is rare, however, that the results come out entirely as expected, and it is necessary to keep a continuous watch on progress and alter the programme as the work proceeds.

Having decided on a system which appears to give an acceptable solution, it is necessary to make a thorough check on the simulator over a wide range of conditions such as altitude, target speed, course and manoeuvre, noise level etc. Many hundreds of runs are required to establish performance over the range of probable operating conditions.

There are thus three stages in system design:—

(*a*) A qualitative study of a particular effect with simplified representation.

(*b*) Detailed simulator study of a selected range of parameters in a sensitive situation.

(*c*) Detailed survey of the chosen solution in a wide range of conditions.

ACKNOWLEDGMENTS

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TECHNICAL NOTES

The Initial Instability of an Elastic Sheet Reinforced by Stringers and Skew Ribs

G. G. POPE, M.Sc.(Eng.)
(*Structures Department, Royal Aircraft Establishment*)

1. INTRODUCTION

This note is devoted to the analysis of the initial buckling of an elastic sheet reinforced by a uniform non-buckling grid of stringers and skew ribs. The stringers and ribs enter the analysis through their influence on the stresses in the unbuckled sheet. Combinations of the following loadings in the plane of the reinforced sheet are considered:—

(*i*) uniform direct loading normal to the ribs,
(*ii*) uniform shear loading relative to axes perpendicular and parallel to the ribs.

The following assumptions are made:—

(*i*) the ribs and stringers do not bend,
(*ii*) the stringers have negligible torsional rigidity, and
(*iii*) either the ribs are not directly attached to the sheet, or they are sufficiently widely spaced for kinematic restraints imposed by them on the out of plane movements of the sheet to be neglected.

NOTATION

Ox, Oy	Cartesian axes, Ox parallel to ribs
OX, OY	Cartesian axes, OY parallel to stringers
a	rib pitch
b	stringer pitch
c	$\cos\eta$
s	$\sin\eta$
t	skin thickness
w	deflection of skin in buckled state
w_0	indeterminate constant
E	Young's modulus
K	$1+R+S+c^2RS(1+\nu)(1+s^2-\nu c^2)$
R	(effective rib-flange area)$/at$
S	(stringer section area)$/bt$
β	parameter used in defining buckling mode
η	angle between OY and Oy
η^*	value of η at which mode shape changes
λ	(wavelength of initial buckles)$/b$
ν	Poisson's ratio (assumed to be 0·3 in calculations)
ϕ	$\dfrac{\pi^2}{12(1-\nu^2)}\dfrac{Et^2}{b^2}$
$\bar\sigma_x, \bar\sigma_y, \bar\tau_{xy}$	(applied loading per unit width referred to x, y axes)$/t$
$\sigma_x, \sigma_y, \tau_{xy}$	stresses in sheet referred to x, y axes
$\sigma_X, \sigma_Y, \tau_{XY}$	stresses in sheet referred to X, Y axes
σ_S, σ_R	direct stresses in stringers and ribs

The above stresses and loads are defined as positive in tension.

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$K_X, K_Y, K_{XY}, \bar K_y$	buckling interaction parameters, where for example

$$K_X = -\,(\text{critical value of } \sigma_X)\Big/\frac{Et^2}{b^2}$$

$$\bar K_y = -\,(\text{critical value of } \bar\sigma_y)\Big/\frac{Et^2}{b^2}$$

2. STRESSES IN THE UNBUCKLED STATE

The axes used in this and the following sections are shown in Fig. 1. Mansfield[1] has derived expressions for the sheet stresses referred to axes along and perpendicular to the ribs. The expressions given below for the sheet stresses referred to axes along and perpendicular to the stringers have been obtained by applying the usual stress transformation formulae to Mansfield's results.

The stresses in the sheet due to a direct load $t\bar\sigma_y$ per unit width applied normal to the ribs are given by

$$\left.\begin{aligned}\frac{\sigma_X}{\bar\sigma_y} &= \frac{1}{K}\{s^2(1+S)+2s^2c^2(1+\nu)^2RS+R(s^2+\nu c^2)\}\\ \frac{\sigma_Y}{\bar\sigma_y} &= \frac{1}{K}\{c^2(1+R)+\nu s^2(R+S)\}\\ \frac{\tau_{XY}}{\bar\sigma_y} &= -\frac{sc}{K}\{1+S+(1-\nu)R+c^2(1-\nu^2)RS\}\end{aligned}\right\} \quad (1)$$

Under a shear load $t\bar\tau_{xy}$ per unit width the sheet stresses are given by

$$\left.\begin{aligned}\frac{\sigma_X}{\bar\tau_{xy}} &= -\frac{2sc}{K}\{1+R+S+c^2(1+\nu)RS\}\\ \frac{\sigma_Y}{\bar\tau_{xy}} &= \frac{2sc}{K}[1+R-\nu S\{1+(1+\nu)c^2R\}]\\ \frac{\tau_{XY}}{\bar\tau_{xy}} &= \frac{1}{K}\{(1+R+S)(2c^2-1)+SR(1-\nu^2)c^4\}\end{aligned}\right\} \quad (2)$$

The stringer and rib-flange stresses under the relevant loads are given by

$$\left.\begin{aligned}\frac{\sigma_S}{\bar\sigma_y} &= \frac{1}{K}\{c^2-\nu s^2+c^2R(1-\nu^2)\}\\ \frac{\sigma_R}{\bar\sigma_y} &= -\frac{1}{K}\{\nu+Ss^2c^2(1+\nu)^2\}\end{aligned}\right\} \quad (3)$$

$$\left.\begin{aligned}\frac{\sigma_S}{\bar\tau_{xy}} &= \frac{2sc}{K}(1+\nu)(1+R)\\ \frac{\sigma_R}{\bar\tau_{xy}} &= -\frac{2scS}{K}(1+\nu)(s^2-\nu c^2)\end{aligned}\right\} \quad (4)$$

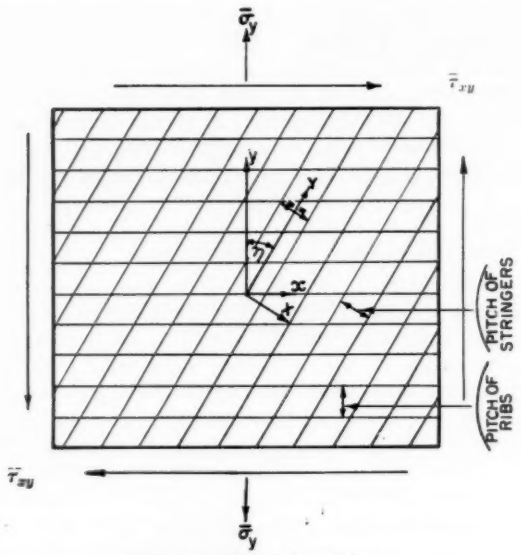

FIGURE 1. General notation.

3. INTERACTION BETWEEN σ_X, σ_Y AND τ_{XY} AT ONSET OF BUCKLING

When σ_X, σ_Y and τ_{XY} have been evaluated, the buckling behaviour of the strips of sheet between adjacent stringers can be analysed. These strips, which are effectively simply supported at their edges, are assumed to be infinitely long. The exact general solution for combined direct and shear loading on an infinite simply-supported strip has been obtained by Schmieden[2], who extended the solution derived for pure shear loading by Southwell and Skan[3]. The computation of the exact solution is laborious and can be circumvented by using approximate methods with negligible loss of accuracy. An approximate general solution, by a standard energy method[4], assuming the waveform

$$w = w_0 \sin \frac{\pi X}{b} \sin \left\{ \frac{\pi (Y - \beta X)}{b\lambda} \right\} \quad . \quad (5)$$

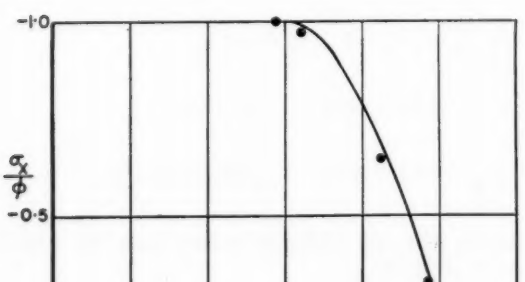

FIGURE 2. Curve illustrating the accuracy of the approximate solution given in this note when $\sigma_Y = 0$.

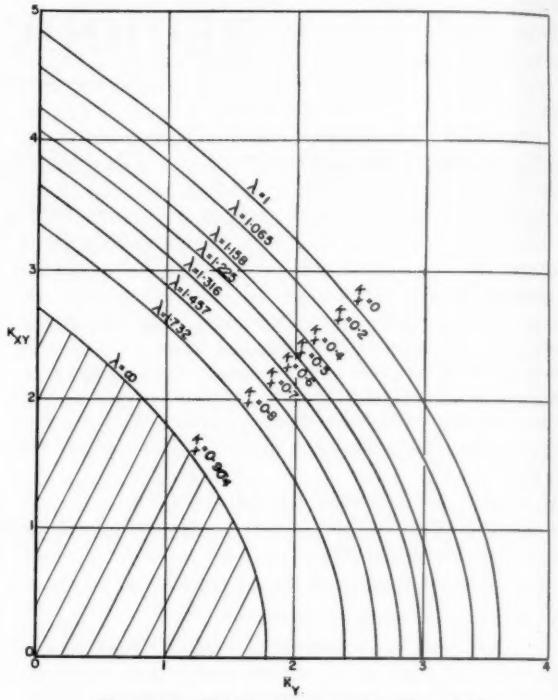

FIGURE 3. Relation between K_X, K_Y and K_{XY}.

yields the following simple expression relating the critical values of σ_X, σ_Y and τ_{XY}.

$$\frac{\tau_{XY}^2}{\phi^2} = 4\left(1 + \frac{\sigma_Y}{2\phi} + \sqrt{\left[\frac{\sigma_X}{\phi} + 1\right]}\right)\left(3 + \frac{\sigma_X}{2\phi} + \sqrt{\left[\frac{\sigma_X}{\phi} + 1\right]}\right) \quad (6)$$

where $$\phi = \frac{\pi^2}{12(1-\nu^2)} \frac{Et^2}{b^2}.$$

The wavelength parameter is given by

$$\lambda = \left\{1 + \frac{\sigma_X}{\phi}\right\}^{-\frac{1}{2}}. \quad . \quad . \quad . \quad (7)$$

Expression (6), which was derived by Wagner[5], is in error by about 6 per cent in the case of pure shear loading. An empirical factor can however be introduced to give the correct result for pure shear loading, without disturbing the result for zero shear, which this method predicts exactly. The resulting expression is

$$\frac{\tau_{XY}^2}{\phi^2} = 3{\cdot}594\left(1 + \frac{\sigma_Y}{2\phi} + \sqrt{\left[\frac{\sigma_X}{\phi} + 1\right]}\right)\left(3 + \frac{\sigma_X}{2\phi} + \sqrt{\left[\frac{\sigma_X}{\phi} + 1\right]}\right). \quad . \quad . \quad . \quad (8)$$

The relationship between the critical values of τ_{XY}/ϕ and σ_X/ϕ when σ_Y is zero, calculated using this expression, is shown in Fig. 2 together with values interpolated from exact results obtained by Schmieden. In view of the accuracy and simplicity of this semi-empirical solution, it is unnecessary to use any more refined approximate method.

Taking Poisson's ratio as 0·3, the buckling condition can be expressed in terms of the K_X, K_Y, K_{XY} notation used in the R.Ae.S. data sheets, giving:—

$$K_{XY}^2 = 2{\cdot}937\left(1 - \frac{K_Y}{1{\cdot}808} + \sqrt{\left[1 - \frac{K_X}{0{\cdot}904}\right]}\right) \times \left(3 - \frac{K_X}{1{\cdot}808} + \sqrt{\left[1 - \frac{K_X}{0{\cdot}904}\right]}\right) \quad (9)$$

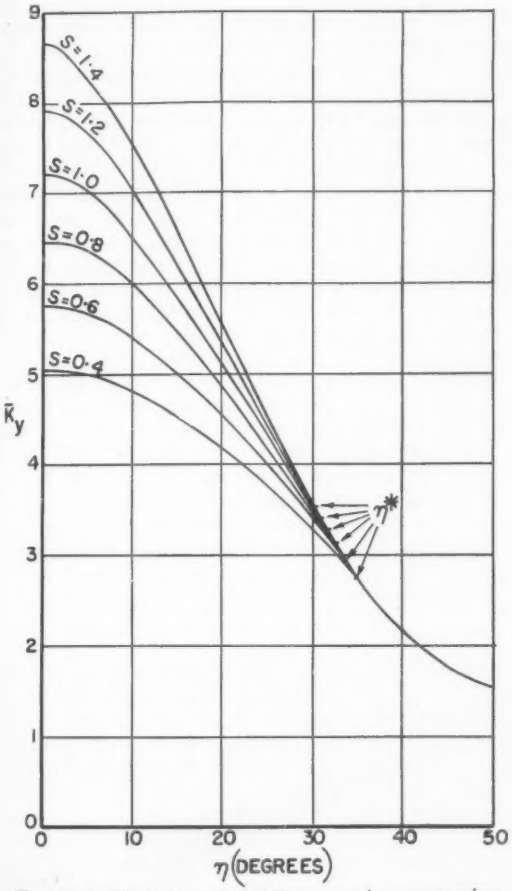

FIGURE 4. Variation of buckling stress in compression with stringer angle.

This expression is plotted in Fig. 3 for positive values of K_X, K_Y and K_{XY}. In the shaded portion of the figure where $K_X=0{\cdot}904$ each strip buckles laterally as an "Euler plate."

4. THE EFFECT OF STRINGER ANGLE ON THE BUCKLING LOAD OF A REINFORCED SHEET

The theory given above is used here to estimate the variation with stringer angle of the buckling load of a sheet reinforced only by stringers, when subjected to a compressive load. The critical values of $\bar{\sigma}_y$ and the corresponding $\bar{K}_y$ are found using equations (1) and (9). $\bar{K}_y$ is plotted as a function of η in Fig. 4 for a range of values of S. When η exceeds a certain value η^*, shown in Fig. 4, K_X has its greatest value and the strips of sheet between adjacent stringers buckle as "Euler plates".

ACKNOWLEDGMENT

The author would like to thank Mrs. C. A. Mason for performing the computations for this note.

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On the Breakdown at High Incidences of the Leading Edge Vortices on Delta Wings

B. J. ELLE

(Saab Aircraft Company, Sweden)

IN A RECENT article, H. Werlé[1], has described how the free spiral vortices on delta wings with leading edge separation suddenly expand if the incidence is increased beyond a critical value. His description conforms to a great extent with the results, arrived at during an English investigation of the same phenomenon (called the vortex breakdown)[2], but the interpretations of the observations, suggested by the two sources, are different. Against this background it is felt that some further comments and some pertinent high speed observations[3], may be of interest.

In both Refs. 1 and 2 it was found that at low speed the breakdown of the leading edge vortices on a 60° delta wing occurs upstream of the trailing edge if the incidence is higher than approximately 11°. Keeping this in mind, the variations of pitching movement with incidence and Mach number for a 60° delta wing (Fig. 1), appear to be particularly interesting and put the low speed observations in a new perspective of greater practical significance.

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Schlieren photographs and surface flow patterns show that the pitch-up occurring at near-sonic speeds at incidences above the dotted line in Fig. 1 is associated with the appearance of a shock wave ahead of the trailing edge and that a vortex breakdown occurs immediately behind this shock wave. It may be tempting to interpret this so that the shock wave is caused by the general pattern of the flow and that the vortex breakdown is a secondary effect, but the critical pitch-up incidence very nearly coincides with the incidence at which the vortex breakdown has been observed at low speed and the apparent continuity of the variation with Mach number of the critical incidence makes it more probable that the shock wave is a direct consequence of the vortex breakdown.

In order to find support for this hypothesis an experiment was made with a flat plate delta wing of 63·4° leading edge sweep. According to Ref. 2, the vortex breakdown on this wing at low speed occurs upstream of the trailing edge if the incidence is above 18° and at high speed it could be expected from Fig. 1 to occur upstream of the

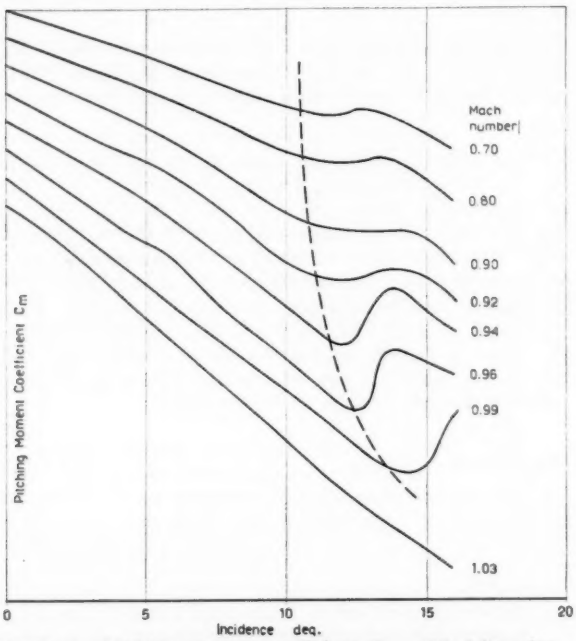

FIGURE 1. Pitching moment coefficient C_m. 60° delta wing.

trailing edge at incidences above 19° or 20°. Fig. 2 shows schlieren photographs of the flow (side view) at incidences, α, of 15°, 18° and 21° and Mach numbers of 0·80 and 0·90. At both speeds the vortex breakdown is visible at $\alpha=21°$ at a position approximately one-third of the root chord upstream of the trailing edge, while at $\alpha=15°$ or $\alpha=18°$, no such phenomenon can be seen. At $M=0·90$ a weak shock wave is seen at the position of the vortex breakdown.

That the breakdown on the photographs is not as distinct as in low speed visualisation is quite reasonable, particularly as the photographs show the superposed pressure gradients around two leading edge vortices. Behind the trailing edge the flow pattern is badly obscured by the presence of the trailing edge vortices and at $M=0·90$ the trailing edge shock wave adds to this.

So far the mechanism of the vortex breakdown has not been touched upon. In Ref. 1 it was suggested that the phenomenon is due to transition from laminar to turbulent flow in the vortex. Although turbulence may affect the breakdown phenomenon at very low Reynolds numbers and the phenomenon apparently is due to some sort of instability, it is improbable that transition is a prime factor. The fact that the observations of Ref. 1 at a Reynolds number of 10^4, (based on root chord) are in perfect quantitative agreement with observations in a number of water

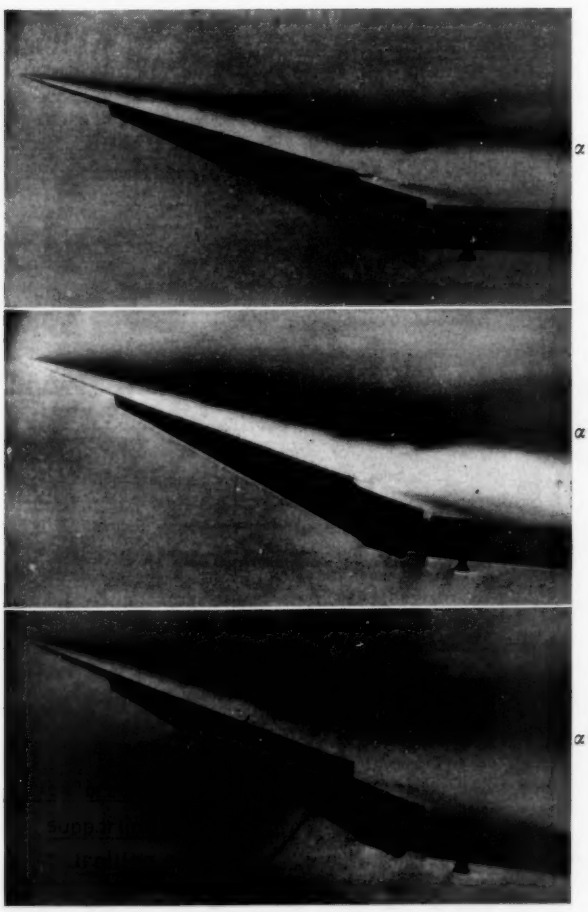

FIGURE 2(*a*). Vortex patterns on a delta wing. Leading edge sweep 63·4°, $M=0·80$.

FIGURE 2(*b*). Vortex patterns on a delta wing. Leading edge sweep 63·4°, $M=0·90$.

and wind tunnels and at Reynolds numbers up to 5×10^6 indicates that something else is the reason.

In Ref. 2 it was suggested that the breakdown may be due to the field of vorticity around the vortex developing in such a way that the downstream transport of fluid in the vortex core fails. From the viewpoint of this hypothesis, which found some support in the low speed experiments of Refs. 1 and 2, the behaviour of the breakdown phenomenon as the speed is increased is by no means surprising. The disappearance upstream of the trailing edge of the breakdown phenomenon if the speed exceeds a Mach number of one is reasonable, as the trailing edge has no effect on the shedding of vorticity from the leading edges at supersonic speeds.

ACKNOWLEDGMENT

The author wishes to thank the Saab Aircraft Company for permission to publish this note.

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Initial Strains in the Matrix Force Method of Structural Analysis

J. H. ARGYRIS and S. KELSEY

(*Department of Aeronautics, Imperial College of Science and Technology*)

1. INTRODUCTION

In a Technical Note in the June JOURNAL (p. 354), Dr. A. L. M. Grzedzielski discusses some aspects of the Matrix Force Method of Structural Analysis and in particular has raised objections to the concept of initial strains and its use by the present writers as an artifice in the treatment of cut-outs and modifications in structures. Dr. Grzedzielski asserts that the idea of initial strains is not valid if the flexibility matrix of the unassembled structural elements is other than purely diagonal. Unfortunately this assertion is just not true. It appears to originate in the excessively narrow view of the analysis of redundant structures inherent in the Castigliano approach adopted in Ref. 1. Neither is the conclusion correct that the method of treatment of modifications which Dr. Grzedzielski presents gives a different result from that of the authors, as is easily shown by elimination of some misprints and completion of the algebra in Ref. 1.

It is of course possible that the objections contained in Ref. 1 have arisen from a misunderstanding of our published work. In what follows therefore, we hope that while discussing in more detail the issues raised by Dr. Grzedzielski we may be able to clarify a little some of the points which may have given rise to misunderstanding. We also hasten to assure Dr. Grzedzielski that, critical or not, his comments call for no apology.

NOTATION

- $\mathbf{S}_i$ column matrix of generalised stresses on a single element
- $\mathbf{v}_i$ column matrix of corresponding generalised strains of a single element
- $\mathbf{f}_i$ flexibility matrix of a single element
- $\mathbf{H}_i$ column matrix of generalised initial strains of a single element
- $\mathbf{b}_1$ rectangular matrix of generalised stresses on all elements due to unit redundancies
- $\mathbf{I}$ unit matrix
- $\mathbf{G}$ column matrix of loads on element to be cut-out (Ref. 1)
- h suffix denoting cut-out or modified elements
- g suffix denoting remaining elements
- $\mathbf{k}$ diagonal matrix defining cross-sectional changes (Ref. 1)
- prime (′) denotes transposed matrix.

2. THE IDEALISED STRUCTURE

We should first like to point out that the Matrix Force Method as presented in Ref. 2 is a general method of analysis applicable, at least in principle, to any kind of structural assembly. As Dr. Grzedzielski himself observes, the successful use of the method for the accurate estimation of the detailed stress distribution in a particular structure depends on a satisfactory idealisation of the structure. However, except in so far as it provides the basic material for the analysis to operate on and providing it ensures a structural system in accordance with the basic assumption of the general theory, the idealisation for a particular case is a distinct and separate process. The concepts and analysis of the general theory are not therefore to be judged by the properties of a particular idealised system.

The basic assumption of the Matrix Force Theory in this connection is that the structural system consists of a finite assembly of discrete elements. The properties of an individual structural element are essentially defined in terms of one or more stress resultants each of which is associated with a simple (known or assumed) distribution of stress within the element. The simplest examples in aircraft structures are the shear flow on a rectangular panel with constant shear stress and the normal forces at the two ends of a flange element carrying direct stress. But, however complicated the element may be, it is assumed that we can analyse it with sufficient accuracy once we know the magnitudes of the chosen stress resultants. It is obvious that the idealisation of the structure as a whole must be consistent, *i.e.* that equilibrium be possible as between the stress resultants on adjoining elements. We regard these stress resultants as the generalised stresses (or simply stresses) of the idealised structure. The deformations of the elements are described by "generalised strains" corresponding to the generalised stresses. These are of two types:

(*a*) those caused directly by the generalised stresses on the elements. For a linearly elastic structure these are given by the element flexibility matrix as $\mathbf{f}_i\mathbf{S}_i$

(*b*) initial strains $\mathbf{H}_i$ caused by thermal expansion or

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incorrect manufacture. Other possible causes are self-equilibrating load systems applied to the element (continuous beams with distributed loading may be handled by this means) or (as Dr. Grzedzielski mentions) plastic straining or yielding. The common feature is that they arise for the unassembled element with zero values of the generalised stresses. These strains have no connection with the flexibility matrix $\mathbf{f}_i$ and cannot therefore be influenced as a concept by whether $\mathbf{f}_i$ is a diagonal or a " full " square matrix.

A necessary step in the analysis of a particular problem is, of course, the calculation of the element strains from the detailed strain distribution within the element, *i.e.* the generalised initial strain as well as the flexibility matrix $\mathbf{f}_i$ for the generalised elastic strain. The relevant tool here is the Unit Load Method in its "continuous" or "integral" form. Ref. 2, Section 5(*c*), illustrates this for a simple case of initial strain and in Ref. 4, Section 26, a related problem in connection with the assembly of a number of simple elements into a single more complex one is discussed. It is true, as Dr. Grzedzielski infers, that in swept wings the panels may contribute initial strains due to thermal expansion. This is certainly not excluded by the general theory and the necessary terms can be calculated by the method as mentioned above.

It may be relevant here to mention that for such structures as swept, low aspect ratio wings the displacement method has, in considerable experience on both sides of the Atlantic, proved the more suitable. The main reason is the inherently simpler analysis of the element properties which is possible with a kinematic idealisation for elements of unusual shape.

3. ANALYSIS OF THE IDEALISED STRUCTURE

Dr. Grzedzielski remarks that the analysis " is based on the Castigliano theorem of least work." This is no doubt a source of misunderstanding. The limitations of the Castigliano method are clearly brought out by the tricks he has to play in order to force his thermal stress analysis into the Castigliano mould. It has to be pretended that the thermal expansion is really an elastic strain caused by stresses due to " fictitious thermal loads." This is all the more incongruous when the stresses themselves must subsequently be dropped from the results and we are left only with the strains they cause, *i.e.* the thermal expansion.

The appeal made to the authority of Timoshenko for the presentation of the basic data of thermal stress analysis as fictitious thermal forces is vitiated by the fact that the analysis presented there is in terms of displacement. In such an analysis—whether matrix or otherwise—we start with compatible systems and form equations for the unknown displacements from the equilibrium condition.

In the Force Method, however, it is the compatibility conditions which provide the equations for our unknown forces (the redundancies). These conditions are *conditions on the total strains* of the elements and hence it is the initial strains—the free thermal expansion or distortion—of the elements which are required as the natural data.

A straightforward application of the Principle of Virtual Forces, or its equivalent—the Unit Load Theorem —yields immediately the matrix compatibility equation for the strains

$$\mathbf{b}_1'\mathbf{v}=\mathbf{0}.$$

In this equation, the origin of the strains $\mathbf{v}$ is immaterial. We only require that they are the total (*i.e.* initial plus elastic) strains of the elements. The two equations (38)[3] and (47)[2] which Dr. Grzedzielski quotes from Ref. 2 are simply two particular forms of this general equation, obtained by substituting the relevant forms for $\mathbf{v}$, and are in fact presented as such in Ref. 2.

4. CUT-OUTS AND MODIFICATIONS

The argument presented by Dr. Grzedzielski for the cut-out analysis is of course correct. It is, however, limited and it is in assuming that this limitation applies also to the initial strain derivation of Ref. 2 that Dr. Grzedzielski falsely concludes that the results in Ref. 2 apply only when the element flexibility matrix is diagonal.

In order to simulate a cut-out in a continuous structure, we have to devise a means whereby the element to be cut out experiences strains while not applying load to the adjoining structural elements. (In the most general case one or more linear combinations of the generalised stresses applied to the adjoining elements have to be zero.) Our argument in Ref. 2 achieves this by the simple expedient of supposing that these strains are initial strains. In other words the element is imagined as so distorted that it fits in with its loaded and strained neighbours while retaining zero values of the particular generalised stresses as required by the cut-out. On the other hand, the method adopted by Dr. Grzedzielski in Ref. 1 is to imagine the necessary strains as caused by externally applied generalised stresses. Compatibility is again ensured while no forces are transmitted to the neighbouring structure. The trouble is that the element flexibility matrix enters now into the picture. Suppose we wish to simulate a cut-out where only some of the generalised stresses on the element have to be zero. It is then only the generalised strains corresponding to these stresses which we are free to alter. But if the flexibility matrix is not diagonal these strains have to be considered as caused by all the generalised stresses. This is why the method as given by Dr. Grzedzielski is limited to the case of a diagonal $\mathbf{f}_i$.

On the other hand, there is no difficulty in supposing arbitrary magnitudes of only some of the *initial* strains quite independently of the others since the flexibility matrix never enters into the argument. The limitation of the method of Ref. 1 arises from the insistence that the strains of the elements, whose elimination or cutting is to be simulated, must be elastic strains under externally applied loads. Although it is possible to get round this difficulty, such complication seems unnecessary since in the argument based on the initial strain concept the difficulty never even arises.

We have been well aware that alternative derivations are possible to the cut-out and modification problem. The above discussion provides one reason for the superiority of the initial strain method. Another is that, as with the thermal stress analysis, it fits most naturally and simply into the general theory.

If it is necessary to carry the physical argument behind the method to its limit, we find no objection to the idea of a Maxwellian demon blowing hot or cold on various structural elements in order to ensure that the relations between stress and total strain on them achieve the desired result. After all it is only necessary that we be able to visualise such a process so as to follow it with our calculations. Moreover our " hot and cold " demons have an easier time of it than Dr. Grzedzielski's agent, who would be most strenuously employed in applying the necessary external force systems to the elements in order to achieve the same results. But these helpful demons are the very stuff of structural analysis—whether they are imposing

virtual displacements, enforcing constraints or making and welding-up the cuts of which we are so fond. The difficulty lies not in visualising them but in devising the right jobs for them. They are in any case much more prosaic fellows than Maxwell's original accomplice in his hypothetical violation of the Second Law of Thermodynamics.

We have already mentioned that alternative arguments are possible for the derivation of the cut-out and modification formulae. But, as Dr. Grzedzielski himself remarks and, providing all approaches are equally general and based on the same system, they should all give the same result. That this is so in the present case is shown by substitution for **G** from equation $(17)^{(1)}$ into $(18)^{(1)}$. (The right hand side of the second of equations $(18)^{(1)}$ should surely read

$$-[\mathbf{I}+\mathbf{k}^{-1}]\,\mathbf{G}.)$$

A little manipulation and combination of the two expressions for $\mathbf{S}_a$ and $\mathbf{S}_h$ yields then the single equation (71) of Ref. 2. (Note also that the terms $\mathbf{b}_1'$ in equations (9), (10), (11), (17), $(18)^{(1)}$ should be $\mathbf{b}_{1h}'$.) Thus it is not necessary to resort to numerical computations to prove that the two derivations lead to the same result.

Actually, equation $(71)^{(2)}$ is only a special case of equation $(68)^{(2)}$ for modified elements which are geometrically similar to the original ones. Equation $(68)^{(2)}$ allows a more general modification which may involve the tapering of elements. The question of whether or not **f** is diagonal does not arise; it is only supposed that a part of it is altered by adding the sub-matrix $\Delta\mathbf{f}_h$. The accuracy and validity of the result is moreover directly verified in Ref. 3 (Section 3.2) by an independent investigation of the compatibility conditions in the modified structure—without reference to initial strains.

Since the question of limitations of the cut-out method has been raised, we ought possibly to point out that this method is directly applicable to any cut-out which is defined by zero values of one or more of the chosen generalised stresses—or a linear combination of them, *e.g.* at a hinge or cut at an intermediate point of a continuous beam element, for which the relevant generalised stresses are the bending moments at the two ends of the element. In Ref. 4, Section 29, it is also shown how the method may be used to "cut" to a prescribed value of a generalised stress in an element. Naturally, some cut-outs in an actual structure are not so simply defined, since they do not fit into the geometry of the idealised structure, *e.g.* circular holes. Their effect on the overall stress distribution in the structure is determined by attributing a suitably increased flexibility to the element containing them.

On the other hand, with large cut-outs extending over a number of elements, it is only necessary to nullify the generalised stresses on the boundary of the cut-out. (It is amusing that some residual stresses—of course self-equilibrating—may remain within the cut-out structure.)

A more general problem is found with modifications and/or cut-outs in elements of a statically indeterminate sub-structure which is taken as a single complex element in the complete structure, *i.e.* we work with a basic system which is itself statically indeterminate. The necessary theory, illustrated by application to the transverse frames in a fuselage, is in Ref. 4, Section 8.

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THE BRANCHES

The fourth extant Branch is Brough originally as Leeds in 1927, becoming Leeds and Brough in 1929 and, since 1936, Brough.

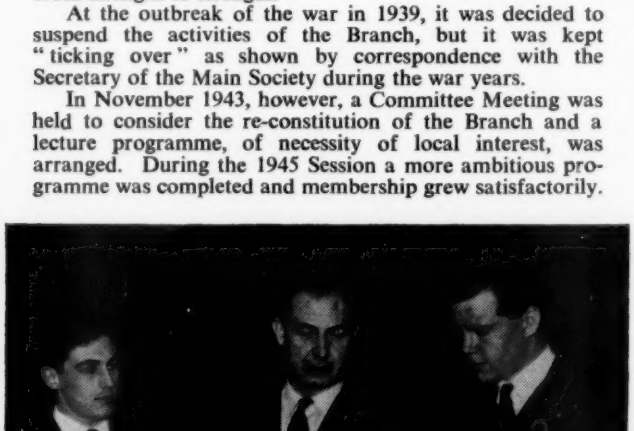
In the February 1926 JOURNAL OF THE ROYAL AERONAUTICAL SOCIETY, it was reported that the Society proposed to form Branches in selected areas of the provinces and, in the issue for April 1927, it was recorded that a Branch had been formed in Leeds. Centred at the University of Leeds it owed its existence in the main to the support given it by the Directors and technical staff of the Blackburn Aeroplane and Motor Co. Ltd., the headquarters of which were at Olympis, Leeds. This early branch had a committee composed of members of the University and Mr. Robert Blackburn and Major Bumpus, Directors of the Blackburn Company.

The opening lecture of the 1927-28 Session was held at the University on 10th November 1927, under the Chairmanship of the President of the Society, Air Vice-Marshal Sir Sefton Brancker, and thus was established one of the earliest of the Society's many Branches.

Gradually the Blackburn Company was transferring its activities to its Seaplane Base at Brough, near Hull, in East Yorkshire and as the technical staff was progressively reduced in Leeds, this inevitably led to a transference of the Branch interests to Brough and the formation of the Hull and Leeds Branch. The initial meeting of this Branch was held on 6th December 1929 at the Aviation Club, Brough. During the early 1930s the Branch had its difficulties and at one point had almost come to a full stop, with barely 50 members, but it managed to keep going and since 1936, when it was formally entitled the Brough Branch, has gone from strength to strength.

At the outbreak of the war in 1939, it was decided to suspend the activities of the Branch, but it was kept "ticking over" as shown by correspondence with the Secretary of the Main Society during the war years.

In November 1943, however, a Committee Meeting was held to consider the re-constitution of the Branch and a lecture programme, of necessity of local interest, was arranged. During the 1945 Session a more ambitious programme was completed and membership grew satisfactorily.



Courtesy of Blackburn "Courier"

Mr. J. A. Miller (left) and Mr. C. J. Wood (right), N. E. Rowe Medal winners, with Mr. A. D. Howarth, Branch Secretary.

In August 1948, a most successful Flying Display was organised in aid of the R.A.F. Benevolent Fund which resulted in £309 being paid into the fund. The following year on 18th June 1949 a similar Display was organised and this time resulted in £306 being paid to the Fund.

The Branch had its first Main Society Lecture in February 1952, which proved a great success. In 1954, the Branch founded the Cayley Memorial Lecture, in memory of Sir George Cayley (1773-1857), a Yorkshireman who is known as "the Father of British Aeronautics." This has been a most distinguished series, beginning with the inaugural lecture by Capt. J. L. Pritchard in which he gave a masterly paper on Cayley with much new information about the man and his inventions. The Cayley Lecture is now the highlight of the Lecture Programme and each year draws a distinguished audience. The Sixth Lecture in November 1959 was by Sir George Gardner, at that time Director of the R.A.E.

To commemorate the 50th Anniversary of Powered Controlled Flight, the Branch held a Dinner at the Hull Guildhall, after which the film "Powered Fight—The Story of the Century" was shown. The Dinner was attended by 168 Members and official guests, who included the President of the Royal Aeronautical Society, Sir William Farren, Mr. Robert Blackburn and many other well-known figures in aviation, past and present.

Since 1948, when there were about 100 Members, the Membership has shown a steady rise and by 31st March 1960 had reached a total of 695 of whom 212 were Main Society Members. This puts Brough among the top four biggest Branches. The growth in Membership has been the result of a strenuous drive for new members, which was described in the February 1959 JOURNAL.

Brough has been fortunate in its Honorary Secretaries and has had only two during the past 15 years—Mr. F. A. Wilkinson from 1945-1956 and since 1956, Mr. A. D. Howarth. The Branch is certainly an energetic one as the activities for 1959-60 indicate. During this Session 9 film shows were held at which the average attendance was 47, 8 visits were made (average attendance 52) and 8 "ordinary" lectures and 9 other lectures, the average attendance at all lectures being well over 100.

An event which has been outstandingly successful is the "Young People's Christmas Lecture," first held in 1958, which has drawn an audience of 600 boys and girls for the past two years.

This year the Branch has achieved the distinction of winning both the N. E. Rowe Medals for 1960. Founded in 1956 to encourage younger members of the Branches and named after Mr. N. E. Rowe to commemorate his work as Chairman of the Branches' Committee from 1946-1955, the Medals are awarded for the best lecture given before any Branch of the Society by a Member of the Branch in two age groups—(*i*) between 21 and 26 years of age, and (*ii*) under 21 years.

The Medals are only awarded if the lectures are considered of sufficient merit. The 1960 prize-winning lectures were:—(Under 21) "A History of the Development of Flight-Refuelling" by J. A. Miller and (over 21) "Transonic Buffeting on Aerofoils" by C. J. Wood. The papers will be published in the JOURNAL.

This is the first time that the Medals in both categories have been won by Members of the same branch in any one year, and the fact that Brough has achieved this must be very gratifying to Mr. N. E. Rowe, Technical Director of Blackburn Aircraft Limited and President of the Branch.

Note

Mr. Wansbrough-White is not responsible for any material on this page this month. Unfortunately he is ill, but hopes to resume his Editorial duties next month. Members of the Branches will, we know, join in wishing him a speedy recovery.—Ed.

Graduates' and Students' Section

The Committee and its Activities

"The Section exists to provide knowledge in all Branches of Aeronautical Science, to assist the successful advancement of Graduates and Students to the higher technical grades of the Society, to secure that the best use is made of all the privileges arranged by the Council of the Society for the benefit of the Graduates and Students, to encourage competition for the Society's prizes and in any way to further the interests of the Graduates and Students. The activities of the Section shall be controlled by the Council through a Committee of Council known as the Graduates' and Students' Committee."

With this as its terms of reference the 1960-61 Committee intends to carry on with the policy of its predecessor by organising lectures, visits, various social activities and to keep under review matters affecting the interests of the members.

The lecture programme this year has, of necessity, been somewhat curtailed, due to the fact that the new Lecture Theatre will not be in use until November; however, the lectures arranged so far are as follows:—

23rd Nov.—British Space Programme: M. O. Robins

14th Dec.—B.O.A.C. and the Future: C. Abel

8th Feb. 1961.—Flight Testing of V.T.O.L. Aircraft: T. W. Brookes-Smith.

Further lectures on Small Car Design, British Missile Defence Policy and Aeroflot are being arranged; the full programme will be published in the Lecture Cards which members receive in September and January.

Pending visits are to R.A.E. Farnborough, Guinness' Brewery, Lotus, and Associated-Rediffusion. Further visits to N.G.T.E., Boscombe Down, Handley Page and a wine cellar are envisaged. In addition, it is hoped to visit the Paris Air Show in June next year. This will be a repeat of the visit made in 1959, when thirty-six Graduates and Students spent a most enjoyable weekend in Paris—it is rumoured that some of the visitors on that occasion actually saw the air show! The total cost of that visit for the flights to and from Le Bourget and two nights hotel accommodation was £10 10s. 0d. a head.

The Annual Dance this year will be held on Friday, 25th November, and the double ticket price of fourteen shillings for food, wine and beer will be maintained The Summer Party will be held in June, 1961. As a result of the success of "The Committee versus The Rest" cricket match further games against the Graduates' and Students' Sections of the other Engineering Institutions are being contemplated.

The Section's page in THE JOURNAL will continue to give summaries of lectures and visits. The new Hon. Editor, Charles Keil, is a professional Aviation Journalist and is as keen as his predecessor to publish letters from members on matters which affect them in their work or in the larger field of the profession in general.

For some time the Section Committee has been pressing the Society to issue a comprehensive and impartial guide to careers in the Aircraft Industry, which would be of the greatest value to those who want to join the Industry. The Council have agreed and it is hoped that this guide will appear in the not-too-distant future.

In an effort to encourage the interests of Graduate and Student members outside the London area the Committee has established representatives in most of the main educational and industrial centres and visits to aircraft manufacturers in the Coventry and Brough areas have been arranged. Other activities in the provinces will depend on the success of future visits in those areas.

Finally, the Committee is most anxious to receive (polite) suggestions from members for further lectures, visits and activities, but it should be remembered that these future ventures are dependent on the success of the established functions, and so it is hoped that there will be a further increase this year in the attendance figures at lectures, visits and dances, etc.—W.G.W.

Report on Cricket Match, Committee *v.* The Rest

Despite appalling weather, with bad light and an intermittent drizzle, the Handley Page Sports Club ground at Cool Oak Lane, Hendon, was the venue for an otherwise enjoyable match on 24th June.

WATCH THE FAST BOWLER – HE'S IN THE GUIDED FLIGHT SECTION

Malcolm Moulton, having spiked the Committee's guns by guessing correctly that they had a double-headed penny, put them in on a wicket which could only be described as wet. Since the conditions were so poor it was agreed that only 10 overs a side would be played, so all the batsmen had to go for runs. The Committee's opening batsmen were Jim Cownie, and an individual who appeared to be a close relative of the great Dr. W. G., complete with large bushy beard and top hat. He had earlier arrived driving the car belonging to Peter Ross, so it was presumed he had stolen it. After the match, the bearded individual was not to be seen, but Peter Ross was. There's a clue there somewhere The Committee's innings was noticeable for some spirited batting by Duncan and Wilby, and 50 runs were scored.

When the teams changed over The Rest's opening pair were confronted with an attacking field with three slips, two leg slips, silly mid-on and off and two short legs. The Committee's opening bowler, Roy Williams, began his run at the boundary line under the sight-screen. Unfortunately, by the time he arrived at the wicket he must have been a bit tired, for the ball, delivered slightly above snail's pace, proceeded fourth bounce to the wicket keeper, with a somewhat surprised batsman making an unavailing swipe at it *en passant*.

The exact details of The Rest's innings are a little difficult to trace as by this time the scorer was getting cold and there was considerable difficulty in telling friend from foe in the gathering gloom. However, the best innings came from Luxton, and the final result was declared to be a diplomatic tie.

Both teams and supporters now retired to the Pavilion, where there was dancing to help warm cold feet and refreshment to warm the inner person.

Raffle prizes were won by Messrs. Cownie, Clark and Burrin and Mrs. Cownie showed she could "twiddle" with the best in winning a "yacht" race.

We now look forward with confidence to meeting a cricket team from one of the Engineering Institutions, if such a fixture can be arranged.—A.H.F.-M.

THE LIBRARY

Reviews

THE THEORY OF THIN SHELLS. *V. V. Novozhilov. Noordhoff, Groningen, 1959. 376 pp. Diagrams. 68s.*

With the increasing number of translations of Russian books on Engineering and Science now becoming available, no excuse can be offered any longer for ignoring the many past and present Slavonic achievements. Our traditional insularity towards Eastern (or for that matter also Continental) technological developments is in fact fast crumbling away. Actually, as far as the theory of elasticity and structures is concerned we always sensed—no doubt influenced by the masterly series of books due to Timoshenko—that there were considerable untapped Russian contributions. This has been brought forcibly to our attention by J. R. M. Radok's translation of Mushelisvili's book on *Some Basic Problems of the Mathematical Theory of Elasticity* which showed all too clearly that parallel Anglo-Saxon and German attempts on the application of the complex variable technique were rather lame affairs. It is for this reason that a translation of Novozhilov's book on *The Theory of Thin Shells* (a standard textbook in Russia) is welcome. Let it be said immediately that it is restricted to the classical linear theory for small displacements which neglects also the transverse shear strains. Solely statical problems under exclusion and any instability phenomena are considered. Keeping these limitations in mind it offers a valuable insight into Russian work on the theory of shells which includes such, by now, well-known authors as Vlasov, Mushtari, Gol'denweizer, Lur'e, Rabatnov, etc. Thus, the book should meet from this point of view the inquisitiveness of any interested reader in the Western World. Unfortunately, it is in many other ways unsatisfactory. First of all, it is served by an awkward and wooden rendering into English which reads strangely in places. Undoubtedly the original Russian text itself is rather uninspiring—which can be seen from the long-winded and often dull arguments offered on pedestrian matters. But, on the other hand, the translator's preface mentions that a sentence by sentence translation of the original had been aimed at. Why this should have been so appears strange to anyone versed in linguistic matters. After all, one cannot do justice to an author and produce a new living text by clinging to that extent to the original.

Apart from this deficiency there are other critical aspects to the book. Thus, one cannot help noting with astonishment, if not with amusement, that while Novozhilov quotes a fairly representative list of pre-war non-Russian contributions to the shell theory, he does not quote once —but for a single and undated reference to Timoshenko's 1940 book on *The Theory of Plates and Shells*—any post-1939 foreign contribution. This appears to be a systematic policy in many post-war Russian textbooks. It inevitably produced an unbalanced presentation but, on the other hand, the English-speaking reader will not in general mind if he is *au courant* of the Western literature. Nevertheless, the translators would have been well advised to add a list of recent Anglo-Saxon papers on the theory of shells.

A final word of criticism concerns the presentation. This will be found by engineers and scientists mainly interested in the physical ideas, as exceedingly dry and unnecessarily mathematical in its argumentation. The mathematical derivations—admittedly always elementary and copiously if not tediously explained—are always in the foreground, the physical and geometrical ideas usually following. Inevitably, this approach, rather typical of a mind which considers the subject as an exercise in applied mathematics, will be found by many readers as wanting in imaginative skill. Although the ultimate standard reached by the book is not so very much higher than that of Flügge's pedagogically more successful treatise, it is much less suitable as a textbook unless one has a fair knowledge of the subject.

However, irrespective of the above reservations, we cannot but be grateful to Messrs. Novozhilov and Lowe for their extensive account of many notable Russian papers on the shell theory. Let us now look more closely at the contents of their book.

Chapter I presents a detailed account of the classical (Aron-Love) small deflection theory of thin shells of arbitrary geometry, in which we find a number of interesting Russian contributions. To establish the theory, use is made of the analytical geometry of arbitrary surfaces referred to the orthogonal curvilinear system of co-ordinates formed by the lines of principal curvature on the surface. A valuable aspect of the presentation is a clear exposition of what terms can be neglected within the initial assumptions of the linear theory of thin shells. Having derived the deflections of the shell in terms of the deformations of the middle surface, the author gives the compatibility equations. The equilibrium conditions for an element are established and following Lur'e and Gol'denweizer it is shown how these can be satisfied identically in the homogeneous case by the introduction of four stress functions; these functions may be considered as a generalisation of the classical Airy stress function in plane elasticity. The deduction of the strain energy of the shell is along familiar lines and leads ultimately to the usual simple expression. Of greater interest are the relations between internal stress resultants and strains of the middle surface which are derived from the strain energy expressions and it is shown that in their most elementary form (*c.f.* Love) they do not correspond, as often assumed, to the most simple strain energy expression. The question of the most appropriate internal force-strain relations is further illuminated through reference to relevant work of the author, Balabukh and Lur'e. Interestingly enough the formulae of Lur'e although in general much more cumbersome than those of the author, reduce to a simpler form in the case of spherical shells. The following presentation of the boundary conditions is the standard one—*i.e.* Kirchhoff's assumption generalised for shells. The same may be said about the re-writing of the compatibility equations in terms of the internal resultants. We then enter, however a lengthy discussion on the formation of the basic differential equations in the unknowns—be they forces or displacements—which is of some interest since it includes some distinct Russian contributions. Thus, following up on the well-known complex transformation introduced by Meissner, the author obtains two coupled partial differential equations in which the unknowns appear in complex form. This halves the order of the differential equations (they are jointly of the 4th order) and may be considered to contribute to their conciseness and simpler analysis. However, this transformation is only exactly feasible for zero Poisson's ratio. For the more realistic case of non-zero Poisson's ratio, additional simplifications have to be introduced which are, moreover, not identical when the equations are expressed in terms of resultants or displace-

ments. The author discusses subsequently an approximate theory of shells in which the effect of the tangential displacements on the changes of curvature and twist are neglected. He ascribes the consistent application of these and associated simplifications to Mushtari and Vlasov, although similar work originated also in the Western world. Essentially, these approximations are permissible when the stresses due to the moments are of comparable (or smaller) order to those due to the membrane resultants. Two coupled partial differential equations of the 4th order in a stress function and the normal displacements are derived which may be combined again into a single complex form.

The Chapter concludes with the usual classification of shell problems. The limiting case of a pure membrane state is introduced and discussed at some length in Chapter II. The reader will find here hardly anything novel and the presentation appears rather lifeless if compared with the corresponding sections in Flügge's well-known book. Having given the fundamental equations of the membrane theory for the internal forces and the displacements, together with the necessary boundary conditions, the author examines carefully the requirements for the existence of the membrane state of stress. There follows a detailed if standard application of the general theory to surfaces of revolution; here it is shown that the partial differential equations of the 2nd order for stresses and displacements are essentially the same. Developing the normal and tangential loadings as Fourier series in terms of the angle of longitude one obtains for each Fourier term a distinct differential equation. The first Fourier coefficients yield symmetrical loading cases for which the differential equations may be integrated to give directly the stresses in the membrane. As an example of this elementary theory, the author presents the analysis of domes loaded under their own weight and compares the internal stresses for a number of shapes. A further example treats the stress distribution of pressure vessels with caps and it is proved that all types of caps discussed cannot fulfil completely the requirements of the membrane theory. The, by now, standard analysis of membranes of uniform strength is also given. Next the author considers wind types of loadings which, as a first approximation, may be taken to be described by the second terms in the Fourier series for the normal and tangential loadings. Here again it is possible to integrate the differential equations and to obtain the general expressions for the internal stresses. For higher order Fourier loadings, for which the direct integration of the differential equations is no longer possible for arbitrary shapes, some space is devoted to the determination of the stresses in membranes, the meridian of which is a curve of second order. Spherical membranes form the simplest application of this class of surface and the author develops in detail the theory for a spherical dome resting on equidistant columns and loaded by its own weight—the reader familiar with Flügge's book will note the much more lucid analysis of the same problem given there. One of the most interesting and aesthetically satisfying techniques of membrane analysis, the affine transformation of Dischinger, is treated fairly comprehensively. There follows a careful investigation of the existence of pure bending deformations of shells of revolution. Two particular cases, that of spherical and catenoid surfaces—being examples of shells with positive and negative Gaussian curvature respectively—are analysed and it is shown that the boundary conditions which secure a membrane state in the former do not in general guarantee it for the latter surface. The final sections of this Chapter deal with membrane theory of cylindrical shells with arbitrary cross-section. The method is applied to tubes under uniform internal pressure, the prescribed and identical boundary conditions at the two ends of the tube being either statically determinate or indeterminate; it will be noticed that redundant boundary conditions of membranes are hardly considered in this book. The Chapter concludes with a short discussion on cylindrical vaults subjected to their own weight.

Chapter III is devoted to the theory of cylindrical shells. An introductory section classifies them first on the basis of the ratio length to average radius of curvature. It is indicated how the various limiting cases investigated under simplifying assumptions by Vlasov, Feinberg, Mushtari and others may be derived from a single partial differential equation in terms of a generalised complex stress variable. The approximations involved in this formulation are of the same order as in the complex theory of shells of arbitrary shape and non-zero Poisson's ratio developed in Chapter I. The presentation is concise and allows the stresses and displacements to be derived simply, once the complex stress function is established. The author next turns to cylindrical shells stiffened by equidistant rings which offer no resistance to twist and to displacements normal to their plane. By spreading the elastic properties of the ring in the longitudinal direction the problem is reduced to that of an orthotropic shell for which two coupled partial differential equations are obtained; in the case of a circular shell these may be combined into a single complex form. We then find a rather compact investigation of circular cylinder shell roofs. Representing the loading by Fourier series, independent differential equations for each loading term are obtained and it is shown that for sufficiently smooth loadings it is accurate enough to take for the particular integral the solution corresponding to membrane theory. Thus, the shell problem need only be investigated for the homogeneous case *i.e.* zero surface loadings. These homogeneous solutions are given both for the stress resultants and displacements in the usual form involving hyperbolic and trigonometric formation. The satisfaction of the boundary conditions on the generators becomes involved if many terms in the Fourier series are retained. For this reason it is suggested that for roofs loaded under their own weight it is sufficiently accurate to take only the first Fourier term. The author indicates then how any boundary conditions along the generators may be satisfied. A numerical example illustrates the application of theory on a doubly-symmetrical shell formed by four smoothly joined circular cylinders under internal pressure.

The general theory for cylindrical shells proposed by the author is applicable both for long and short shells. However, for long surfaces the author suggests further simplifications which, although presented in mathematical form, essentially involve a neglect of the longitudinal bending of the shell surface. The approximations yield very simple expressions for the stiffnesses of two straight edges. Comparison of the results for a particular example obtained for this technique with a more accurate analysis due to Gol'denweizer shows good agreement not only for the stiffnesses but also for the case when the roof rests on two elastic beams.

The theory of orthotropic cylindrical shells mentioned in the last but one paragraph is applied next in some detail to roof shells in the form of a circular arc. The solution follows the same lines as in the case of the isotropic roofs discussed previously. The author then turns his attention to isotropic cylindrical tubes with doubly-symmetrical but otherwise arbitrary cross-section and freely supported ends; the loading consists of an internal pressure not necessarily

constant. Expanding in the fundamental differential equation in the complex variable, both the latter and the internal pressure in Fourier series in the longitudinal direction, an ordinary differential equation is obtained for each complex variable coefficient and corresponding loading term. Now, each complex variable term is in its turn expanded in Fourier series in the circumferential direction, the coefficients of the expansion being determined from a set of simultaneous equations derived by application of the Galerkin technique. A numerical example with a continuous variation of the radius of curvature shows good agreement with experimental results. The more complex problem of a doubly-symmetrical tube with arbitrary end conditions and under uniform internal pressure is also tackled. Here the complex variable can evidently only be expanded in the circumferential direction; the complex coefficient of this series must then be determined from a set of simultaneous ordinary differential equations together with the appropriate boundary conditions. The last section is of considerable interest and deals with the semi-membrane theory of cylindrical shells which is credited to Vlasov. It is well known that for cylindrical shells of non-circular cross-section the existence of the membrane solution does not only depend on the boundary conditions, shape, loading, but also on the length. With increasing length the membrane distribution is no longer feasible and it is necessary to consider also the bending deformations. Here an approximate method is applied which neglects the longitudinal bending of the surface and assumes that the lengthwise variation of stresses and strains is negligible in comparison with their crosswise variation.

The last Chapter, devoted to shells of revolution, starts with a historical outline of the subject which once more refers only to Russian contributions as far as the past 20 years or so are concerned. The author remarks correctly that there is little sense in seeking exact solutions which contain terms of an order smaller than that neglected in the derivation of the original differential equation. The asymptotic integration method is hence fully justified. Using a similar transformation to that applied in Chapter II, two simultaneous differential equations in two complex variables are obtained which for zero thickness reduce to the fundamental equations for the membranes of revolution. The solution of the shell problem may be sought by expanding the loading, and the complex variables into peripheral Fourier series by which means we obtain two coupled ordinary differential equations for each set of Fourier terms; corresponding differential equations may be obtained for the displacements. For symmetric deformations of a shell—which are associated with the first terms of the Fourier series—one of the complex variables may be obtained by direct integration and is, in fact, real and identical to the complex variable in the membrane theory, while the second one must be obtained from a second order ordinary differential equation, the particular integral of which may be taken to be approximately equal to that indicated by membrane theory. A discussion of the differential equations for the displacements shows the connection between the complex transformation used by the author and the original one due to Meissner. The solution of the homogeneous differential equation for the second complex variable is then obtained by asymptotic integration and a number of formulae are given with the corresponding stresses and deformations; it is shown that the bending stresses are in general localised near the edges. Using these results, the author derives subsequently the influence coefficients for a dome or cap loaded at the edge by uniform radial forces and moments. These formulae are evidently useful for the solution of the interaction problem between a dome and a supporting structure; the particular example of a long circular cylinder joined to an end cap with an arbitrary meridian illustrates this very well.

The proposed analysis of shells of revolution subject to symmetric deformations breaks down when the particular integral cannot any longer be taken as per membrane theory and/or the asymptotic integration is not sufficiently accurate for the solution of the homogeneous equation. As an example for all such cases the author investigates toroidal shells under uniform internal pressure. He shows first how through an approximation to the non-linear coefficient in the differential equation for the complex stress variable the latter can be expressed in terms of Bessel functions. The particular integral is then obtained by expanding it in the form of a Fourier series. The results find application in the involved problem of a long cylinder with an end cap built up by part of a toroidal shell and a smoothly joined spherical segment.

As in the investigation for membranes in Chapter II, the second terms of the Fourier series of the loading may be considered to represent a wind type of loading. The integration of the two coupled fundamental partial differential equations is reduced ultimately to that of a single ordinary differential equation; neglecting, moreover, a term which is in general small, the author shows that this differential equation is in fact identical to that of the symmetric case. Hence, the approximate integration proposed for the latter can be taken over for the wind loading almost without modification. For higher Fourier loading terms no such simplifying approach can be indicated immediately. However, for certain groups of shells which include spheres and catenoids, it is possible, subject to the usual assumptions, to reduce the integration of the coupled complex differential equations to that of two independent differential equations, one of which is analogous to the equation for the symmetric deformations, while the other coincides with the equation of the membrane theory. The particular case of spherical shells under edge loads is then treated in some detail using once more the two initial differential equations in the two complex variables. One is found to contain only one unknown and to be integrable in terms of spherical harmonics. However, for practical purposes it is preferable to apply the asymptotic method of integration. Introducing the result into the other differential equation, its solution can be written down without difficulty and it is shown that the function for the homogeneous case is obtained simply by superposition of the solutions corresponding to membrane theory and to an edge type of loading. This conclusion is exact within the initial assumptions for spherical shells but not for other types of shells where it follows only from a number of further approximations. The author discusses next, briefly, the analysis of catenoid shells—which are surfaces of negative Gaussian curvature—and proves that the two independent differential equations derived in the last but one section may in the present case be integrated in terms of elementary functions.

The concluding section of the book deals with arbitrary shells of revolution subject to any type of Fourier loading. The approximate method of solution is suggested which goes back to Love who, however, did not discuss the errors involved. The author shows that the reduction of the problem to two independent differential equations as carried out previously for the special class of shells, may again be applied if a number of terms are omitted.

The review of the contents has been presented fairly extensively since, with the exception of Timoshenko's book

on plates and shells (now published in an excellent second edition), there is hardly any textbook in English devoted exclusively to shells. The interested reader who has studied Timoshenko's treatise and is able to follow Flügge's German text, will find no doubt in the distinct Russian ideas a valuable extension of his knowledge on the general theory and the various approximate theories for shells. For all the shortcomings mentioned initially, Novozhilov's book can hence be recommended to all structural engineers interested in the theory of continua.—J. H. ARGYRIS.

THE BEAUTY OF GLIDING, *a collection of photographs, introduced by Philip Wills. Parrish, London, 1960. 95 pp. 35s.*

The introduction includes descriptions of three "big-game" flights by Philip Wills: a wave flight in England in 1954, a climb to 30,000 ft. in New Zealand and a flight in the French "Mistral" in 1956. Exciting flying beautifully described and, on this occasion, accompanied by four lovely air photographs of snow-covered Mount Cook and cloud seas.

The rest of the book consists of 88 sepia photographs covering most aspects of the sport of gliding. Philip Wills, Harry Hensser, Charles Brown, and Sally Anne Thompson are the main contributors. The subjects include many air-to-air shots of sailplanes set against well-chosen cloud backgrounds but the one I liked best was Charles Brown's head-on view of a Tiger Moth struggling with an Eagle sailplane on tow. Sailplanes and clouds are certainly photogenic to most people and looking at these pictures will give great pleasure.

Colour has had to be ignored—the price is already high—but every picture teems with interest. The plates are not titled except in the list at the beginning of the book. Here I found the titles unsatisfactory. "En route to cloudland" does not convey more than the photograph and there are many such cryptic titles. Others are little essays of 20 - 30 words. The reader should have details to satisfy his curiosity, if possible. Number 46 is "Lady in flight"; why not "Mrs. Ann Welch in Gull IV"? Philip Wills and David Ince are identified when they appear but not so the smiling figure in the sacking hat, Tony Goodhart (NATO security measure?). Perhaps these details, like the real name of the Mona Lisa, are unimportant. It is the beauty that counts and that is here in plenty.—A. H. YATES.

GREAT AIRCRAFT. *Norman Macmillan. Bell, London, 1960. 304 pp. Illustrated. 21s.*

Although well-written and containing a good deal of effectively presented historical material, *Famous Aircraft* will be a disappointment to many who have felt the need for such a book. The ten aircraft selected are a surprisingly ill-assorted collection and—if they are intended to be representative of their generations—leave the reader wondering why some have been included at all. Wing Commander Macmillan has chosen: the original Wright Flyer, the Sopwith Camel, the Vickers Vimy, the Parnall Pixie, the Fairey Fox, the Supermarine Spitfire, the Boeing B-29, the English Electric Canberra, the Vickers-Armstrongs Viscount and the de Havilland Comet. Few would argue about the Flyer and the Spitfire and possibly the Camel, B-29, Viscount and Comet, but there is likely to be less agreement about the Vimy and Canberra, still less about the Fox and surely no votes at all for the Pixie.

Another surprising feature of this book is the lack of balance in the amount of space devoted to the various types. 75 pages are allotted to the B-29, 40 to the Vimy, 39 to the Comet, 38 to the Wright Flyer, 25 to the Camel, 14 to the Viscount, 13 each to the Fox and the Canberra, 12 to the Pixie and only 11 to the Spitfire. It is hard to see why more than a quarter of the book should be devoted to the B-29 while the Spitfire, which is surely as famous, rates less than four per cent. That the obscure and quite insignificant Pixie should receive more attention than the Spitfire seems entirely illogical and inexcusable. One is forced to the perhaps rather uncharitable conclusion that the author had various "write-ups" about a selection of aircraft which he had prepared at different times for widely assorted purposes and that he has used them up by combining them without much attention to the way in which they fit together. Even so, the result is extremely readable and few will regret that the altogether excellent account of the part played by the B-29 in the Pacific War has at last found its way into print. The book is worth buying for that part alone.—P. W. BROOKS.

LAPLACE TRANSFORMS FOR ELECTRONIC ENGINEERS. *J. G. Holbrook. Pergamon Press, London, 1959. 259 pp. Diagrams. 50s.*

The term "Laplace Transforms" covers a wide field of mathematical techniques which are, of course, valuable in a number of different disciplines but are of particular interest to the electronic engineer because of their help in general network theory and in the treatment of the special waveforms, such as pulse trains, that have become so prominent in recent years. There is sufficient justification here for the preparation of textbooks such as this in which the presentation, examples and problems are in the language and symbolism with which the electronic engineer or student is most at home.

Mr. Holbrook has written for a self-study reader who has not only a good grasp of circuit analysis but also a thorough grounding in both branches of the calculus and in determinants. Any such reader will find that the author has given him a clear and useful guide, frequently enlivened by a nicely informal style. The first fifty-four pages are devoted to complex-variable fundamentals, Fourier series and the Fourier integral. The direct Laplace transform is derived from this integral on page 55 and the remainder of the text proceeds as follows:— Chapter III: Laplace transforms of common functions, of a derivative and of an integral; Chapter IV: inverse transforms (a departure being made by the author—see page 83—in his emphasis on the inversion integral); Chapter V: theorems of value in operating with the transforms; Chapters VI-VIII: applications to network analysis, special waveforms, Butterworth filters, etc. There are 46 pages of transform tables and an adequate index.

The reviewer does not like the meaning attached by the author to current direction (pages 71 and 155) but perhaps this means that he (the reviewer) is an old-fashioned reactionary. It is perhaps also a matter of taste whether the author should or should not be expected to have provided more problems and should or should not have supplied answers to all of them. Reviewer's verdict:— Recommended to all electronic engineers and students with the above-mentioned qualifications and with the willingness to work hard for an insight into the field.—L. W. MEYER.

SYMPOSIUM ON GROUND EFFECT PHENOMENA. *Princeton University, 1959. 361 pp. Illustrated. No price quoted.*

Much credit should go to the Department of Aeronautical Engineering of Princeton University for not only arranging a symposium on this highly topical subject but for issuing the Proceedings, even without discussion, with such commendable promptness. Of the thirty papers most are American, though there are papers from France, Finland, Canada, Switzerland, and two on the Hovercraft, whose cross-Channel journey is vividly described. It is surprising how much work has already been done on the theory and practice of ground effect machines (GEMs in the States) and in how many places; the David Taylor Model Basin in Washington seems to be well in the lead in this respect and has provided a good summary of its research programme.—E.C.P.

Additions to the Library

Advances in Cryogenic Engineering. Volume 5. K. D. Timmerhaus (*Editor*). Plenum Press, New York. 1960. 584 pp. Illustrated. 13.50 dollars. *To be reviewed.*

Aeronautical Dictionary. Frank Davis Adams. U.S.G.P.O. Washington. 1959. 199 pp. 1.75 dollars. 15s. 9d. from H.M.S.O *Reference only.* First intended as an extension of N.A.C.A. Report 474 (Nomenclature for Aeronautics) but it was soon appreciated that as the Report was last revised in 1933 something more elaborate was needed. Much work has gone into the dictionary and many authorities consulted. An unusual feature is a lengthy bibliography. A few diagrams are included.

Altimetry and the Vertical Separation of Aircraft. International Air Transport Association. I.A.T.A., Montreal. 1960. 24 pp. No price. Summarises the technical findings of the various investigating bodies (I.A.T.A., I.C.A.O. and so on) and the conclusions reached by the Airlines. The contents are based primarily on the work initiated by the I.A.T.A. Technical Committee and subsequent development work by the I.C.A.O. Panel on Vertical Separation of Aircraft. Intended for aviation authorities, aircraft and instrument manufacturers and Member Airlines of I.A.T.A.

Avionics Research: Satellites and Problems of Long Range Detection and Tracking. E. V. D. Glazier *et al.* Pergamon, Oxford. 1960. 257 pp. Illustrated. 63s. *To be reviewed.*

Chemistry of Propellants, The. S. S. Penner and J. Ducarme. Pergamon, London. 1960. 651 pp. Illustrated. 70s. *To be reviewed.*

Controls for Outer Space. P. C. Jessup and H. J. Taubenfeld. Columbia University, New York. (O.U.P. London). 1959. Illustrated. 379 pp. The first sputnik and the first American satellites were launched under the co-operative programme of the International Geophysical Year. This form of international co-operation through scientific bodies may be supplanted or supported by inter-governmental action through the use of the United Nations. The General Assembly passed a resolution in December, 1958, setting up a committee to study the problems of outer space and to see how the United Nations and its various agencies could be utilised. This book considers these problems, analysing the things that need to be done and the possibility that the United Nations and other international organisations might be useful in helping to find solutions. The first of the new "Columbia Studies in International Organization".

Design and Performance of Gas Turbine Power Plants. High Speed Aerodynamics and Jet Propulsion, Vol. XI. W. R. Hawthorne and W. T. Olson. Oxford University Press, London. 1960. 562 pp. Illustrated. £5 5s. 0d. *To be reviewed.*

Effects of Nuclear Radiation on Men and Materials. T. C. Helvey. Chapman and Hall, London. 1959. 56 pp. Illustrated. 15s. This is a small book dealing with "A Few Basic Concepts" (radiation, electron, proton, neutron, alpha and beta particles and so on, a page or less each) "Radiation Effects on Men"; "Radiation Effects on Materials" and "Shield Configuration". It is intended to help the designers of nuclear propelled vehicles rather than those who are in trepidation of the effect of fallout.

Gas Turbine Progress Meeting—Proceedings. U.S. Office of Fuels, Materials and Ordnance. U.S.G.P.O. Washington. 1959. 233 pp. Illustrated. 11s. 3d. from H.M.S.O. The proceedings of a meeting sponsored by the Director of Defense Research and Engineering to provide an opportunity to present information on the current status and expected progress of the gas turbine. Each speaker's contribution is described as a "Presentation".

Glossary of Terms Commonly Used in Aviation Medicine, A. Agard Aero Space Medical Panel. 1959. 30 pp. No price. Agard probably provides the finest ground of all time from which technical glossaries and dictionaries can grow. This takes the form of English translated into French with the English expressions in alphabetical order and an alphabetical list of the *mots français* at the end.

Grundlagen der Analysis. E. Landau. (Third edition). Chelsea Publishing Co., New York. 1960. 173 pp. 1.95 dollars. A first rate book from which to learn mathematical German. A German-English vocabulary is supplied to a text on the fundamentals of mathematical analysis and the theory of numbers and the German is intentionally simple. The first edition was published in Leipzig and all subsequent ones in New York.

Hydrodynamics of Oceans and Atmospheres. C. Eckart. Pergamon, Oxford. 1960. 290 pp. Diagrams. 63s. A systematic introduction to the hydro-dynamics of the earth's atmosphere and oceans, under the action of gravitational and Coriolis forces. A major topic is the effect of the density gradients which exist in the atmosphere and oceans on the motions of which they are capable. A second problem—the effect of the Coriolis force—also receives extensive consideration. The approach to problems of meteorology and oceanography is that of a mathematical physicist using the modern methods of atomic theory.

Internal-Combustion Engine in Practice, The. Volume I. Thermodynamics, Fluid Flow, Performance. C. Fayette Taylor. John Wiley, New York (Chapman and Hall, London). 1960. 574 pp. Diagrams (Charts in pocket). £6 8s. 0d. *To be reviewed.*

Lectures on Ergodic Theory. P. R. Halmos. Chelsea Pub. Co., New York. 1960. 99 pp. 2.95 dollars. Based on lectures in a course in ergodic theory given at the University of Chicago, where the author is Professor of Mathematics.

Radargrammetry. D. Levine. McGraw-Hill, London. 1960. 330 pp. Illustrated. 93s. *To be reviewed.*

Smirnoff Story, The. A. R. Coupar. Jarrolds, London. 1960. 271 pp. Illustrated. 25s. *To be reviewed.*

Some Mathematical Methods of Physics. G. Goertzel and N. Tralli. McGraw-Hill, London. 1960. 300 pp. 66s. Designed primarily for physicists, this book presents a common set of mathematical techniques useful in such fields as quantum mechanics, acoustics, electromagnetic theory, reactor physics, and so on. It is the outgrowth of material developed by the authors for graduate courses at New York University. No attempt is made to cover the entire field of mathematical methods of physics. The authors seek to give non-mathematicians a well-rounded and thorough indication of the basic concepts involved in the study of linear operators, vector spaces, orthogonal functions and Fourier analysis, Green's functions as tools for the solution of physical problems. Brief treatments are included on perturbation, variational, and numerical methods.

Strength of Metal Aircraft Elements. U.S. Department of Defense. U.S.G.P.O., Washington. 1959. 251 pp. £1 2s. 6d. As Military Handbook No. 5 this replaces ANC-5 and is now the responsibility of the Department of Defense and the Federal Aviation Agency. Copies may be ordered through H.M.S.O. at approximately the price given. The U.S. price is 2.50 dollars. Eight chapters (tab-indicated) cover General, Steel, Aluminum, Magnesium, Titanium, Heat Resistant Alloys, Miscellaneous Metals and Alloys and Joints. There are six pages of references.

Systems Preliminary Design. J. J. Jerger. Van Nostrand, New Jersey. 1960. 625 pp. Diagrams. £5 10s. 0d. *To be reviewed.*

Teoria Elicopterului. A. Marinescu. Editura Academiei Republicii Populare Romine, Bucharest. 1960. 267 pp. Diagrams. No price quoted. A monograph on helicopter theory written, presumably, in Rumanian. A synthesis of previous papers by the author and others, the book covers rotor aerodynamics, propeller theory, helicopter performance and stability, the stressing and flutter of rotors and the vibration of other helicopter components, particularly ground resonance.

Udet: A Man's Life. Hans Herlin. Translated by Mervyn Savill. Macdonald, London. 1960. 256 pp. Illustrated. 25s. *To be reviewed.*

Universe of Light, The. Sir William Bragg. 1.85 dollars. **Radiative Transfer.** S. Chandrasekhar. 2.25 dollars. **Treatise on Gyrostatics and Rotational Motion.** Andrew Gray. 2.75 dollars. **Three Copernican Treatises.** Edward Rosen. 1.75 dollars. **Theory of Flight.** Richard von Mises. 2.85 dollars. These five books are all reprints by Dover Publications, New York whose agents in this country are Constable. All titles are familiar to workers in their respective fields and their reprinting will be welcome. The book by Bragg is an expansion of lectures he gave in 1931 at the Royal Institution. It was originally published (in this country) by Bell. *Radiative Transfer* is described as the definitive work in the field. The author (at the University of Chicago) presents the subject in plane-parallel atmospheres as a branch of mathematical physics. Gray's book was first published in 1918. It has contemporary application in the field of missiles and guidance systems. The three Copernican treatises consist of Copernicus's *Commentariolus* his *Letter against Werner* and Rheticus's *Narratio prima*. To the original 1939 edition has been added an 877 item bibliography covering the writings on the subject from 1939 to 1958. *Theory of Flight* is "almost unsurpassed as a balanced, well-written account of fundamental fluid dynamics. . . . may still be recommended for a clearer presentation than is to be found in many recent works, though. . limited to situations in which air compressibility effects are unimportant."

Upper Atmosphere, The. H. S. W. Massey and R. L. F. Boyd. Hutchinson, London. 1960. 333 pp. Illustrated. 63s. *To be reviewed.*

American Society for Testing Materials 1960 Preprints

63 Relation between rolling-contact fatigue life and mechanical properties for several aircraft bearing steels. E. V. Zaretsky and W. J. Anderson.

64 Elevated-temperature tensile and fatigue behavior of unalloyed arc-cast molybdenum. G. W. Brock and G. M. Sinclair.

65 Fatigue properties of some high-strength steels. H. E. Frankel *et al.*

67A Prediction of acoustic fatigue life. H. C. Schjelderup.

67B Fatigue of structural metals under random loading. A. M. Freudenthal.

76A Effect of stress concentration on tensile strength of titanium and steel alloy sheet at various temperatures. G. Sachs and J. G. Sessler.

76B Sharp-edge-notch tensile characteristics of several high-strength titanium sheet alloys at room and cryogenic temperatures. G. B. Espey *et al.*

76C The effect of carbon content on the notch properties of 43XX-vanadium modified and 5 per cent chromium sheet steels. E. P. Klier.

79 Structural beryllium. B. B. Muvdi.

N.B. Requests for these items should carry the prefix ASTM.

INTERAVIA ABC 1960—A CORRECTION

It was stated in the notice given to "Interavia ABC 1960" in the July issue that there was an increase in price of 14s. The price is, in fact, the same (£4) as it was in 1959. The mistake was due to confusion with pre-publication price. Any inconvenience caused is regretted.

Reports

AERODYNAMICS

BOUNDARY LAYER—*see also* INTERNAL FLOW
THERMO-AERODYNAMICS

Boundary-layer stability diagrams for electrically conducting fluids in the presence of a magnetic field. V. J. Rossow. N.A.S.A. T.R. R-37. 1959.
Neutral stability curves pertaining to a two-dimensional infinitesimal sinusoidal disturbance are presented for the laminar flow of an incompressible, electrically conducting fluid over a semi-infinite flat plate in the presence of either a coplanar or transverse magnetic field.—(1.1.2.1×1.4.4.).

Dissociative relaxation of oxygen over an adiabatic flat plate at hypersonic Mach numbers. P. M. Chung and A. D. Anderson. N.A.S.A. T.N. D-140. April 1960.
By means of the integral method, a solution is obtained, with relative ease, to a problem involving finite rate dissociation and recombination in a hypersonic laminar boundary layer. Numerical examples show that dissociative equilibrium is not usually reached over plates of reasonable length. On the basis of the present analysis, heat transfer to a highly cooled plate is discussed.—(1.1.1.4×1.9.1).

An analysis of nose ablation for ballistic vehicles. L. Roberts. N.A.S.A. T.N. D-254. April 1960.
Expressions are presented from which may be calculated the total mass loss experienced by the nose of the vehicle at both the stagnation point, where laminar flow prevails and at the sonic point, where the flow is considered turbulent. A typical calculation shows the change in shape of a hemispherical nose during re-entry.—(1.1.2.4×1.9.1×25).

A method of calculating turbulent-boundary-layer growth at hypersonic Mach numbers. J. C. Sivells and R. G. Payne. Astia Doc. A.D.-208774. March 1959.
A method is developed for calculating the growth of a turbulent boundary layer at hypersonic Mach numbers. Excellent agreement with experimental results from axisymmetric nozzles has been obtained. A new empirical relation is used for the incompressible friction coefficient which agrees with experimental data over a Reynolds number range from 10^5 to 10^9.—(1.1.3.4).

Transition measurements and the correlation of transition sensitive data. J. L. Potter et al. *Astia Doc. A.D.-208775. Feb.* 1959.
Measurements of transition location on typical supersonic bodies are shown to demonstrate the influence of several factors affecting transition. Among these are roughness elements, pressure gradients, and factors associated with changes in unit Reynolds number. Transition was detected by four methods, including hot-wire anemometry, and results are compared.—(1.1.2.4).

COMPRESSIBLE FLOW—*see also* WINGS AND AEROFOILS

A study of equilibrium real-gas effects in hypersonic air nozzles, including charts of thermodynamic properties for equilibrium air. W. D. Erickson and H. S. Creekmore. N.A.S.A. T.N. D-231. April 1960.
Flow parameters given in chart form and calculated for air as a real gas in chemical equilibrium for a series of stagnation temperatures between 2,790°R. and 4,950°R. and pressures up to 1,000 atm. are presented. From these charts it is possible to calibrate a hypersonic nozzle in the conventional way. A method is shown for calculating an inviscid nozzle contour including real-gas effects. Charts of thermodynamic properties for air from which these calculations were made and from which additional calculations may be made are included. Also presented is the limiting Mach number for condensation as a function of stagnation temperature and pressure.—(1.2.3.1×1.5.4×1.12.1.3×32.2.2).

Operational method of determining initial contour of and pressure field about a supersonic jet. G. W. Englert. N.A.S.A. T.N. D-279. April 1960.
Simple expressions for estimating the initial contour of a jet exhausting into a supersonic stream and into quiescent surroundings have been developed by using a modified Laplace transform to solve the linearised potential flow equation. The initial slope of the jet boundary was kept as a parameter in the final expressions, which permitted a study of the use of an exact, in place of a linearised, initial value. Results were compared with characteristics solutions and experimental data. An expression developed to determine the pressure field of a supersonic stream surrounding the jet was applied to determine

the pressure influence of the jet on a nearby flat plate. Results were compared with experiment.—(1.2.3.1×1.5.1).

CONTROLS—*see* LOADS
EXTRA-ATMOSPHERIC TECHNOLOGY

FLUID DYNAMICS—*see also* BOUNDARY LAYER
LOADS
WINGS AND AEROFOILS

The upper atmosphere and satellite drag. R. J. Stirton. Smith. Cont. Astro. Vol. 5. *No.* 2. 1960.
The drag on a sphere moving in the regions of the upper atmosphere is investigated. Calculations show that the drag coefficient departs from the classical value of 2 for spheres in circular orbits of altitude greater than 10^3 km. The drag coefficient may well become much greater than 2 at sufficiently high altitudes.—(1.4×8.1×32.2).

The icing of delta wings of unity aspect ratio having leading edge separation. C. K. Rush. N.R.C. Report L.R.-268. *Dec.* 1959.
Tests were made in an icing wind tunnel of two models, and the resulting ice formations are described and illustrated. The significance of scale is discussed, and it is shown that the tests are representative of icing conditions with large droplets and high speeds.—(1.4×1.10.0.3×5.1).

INTERNAL FLOW—*see also* COMPRESSIBLE FLOW

Sur l'écoulement d'un jet entre parois. — R. Curtet. Pubs. Sc. et Tech. 359. 1960. (*In French*).
Theoretical and experimental studies of the mixing of a jet of incompressible fluid with an ambient fluid of the same nature and a theoretical approach to jets (plane or circular) in a confined space are considered. Integration of a differential system for different cases, with or without velocity of the ambient fluid and at constant height of mixing chamber are described. Experimental results are compared with those calculated for plane flow, allowing for the boundary layer on the walls of the mixing chamber. Recirculation of a round jet in a cylindrical chamber is also included.—(1.5.1).

Écoulement de l'air a grande vitesse dans un tuyau de section circulaire. R. Depassel. Pubs Sc. et Tech. 360. 1960. (*In French*).
Flow conditions at subsonic speeds in appreciably adiabatic conditions are studied. The flow is envisaged as having at least three successive régimes—laminar boundary layer, a turbulent boundary layer with a power law velocity profile (having a variable exponent along the flow) and an established flow. The method of Young and Winterbottom is improved. The theory is compared with experimental results.—(1.5.1.1×1.1.2.2).

Singularitätentheorie der Flügelgitter. L. Meyer. Inst. fur Aero. Zurich. Mitt. 26. 1959. (*In German*).—(1.5.4.1).

Experimentelle Untersuchungen an geraden und gekrümmten Diffusoren. H. Sprenger. Inst. fur Aero. Zurich. Mitt. 27. 1959. (*In German*).
Systematic experimental investigations were carried out to determine the influence of the frictional boundary layer on the incompressible flow in channels with pressure rise along the axis. The forms of diffuser are similar to the draught tubes of water turbines and diffuser passages in pumps and blowers. In the region where the conversion of flow velocity into pressure begins, the flow conditions were investigated in detail with the aid of a short precision diffuser. Comparative measurements were also performed with a straight conical diffuser of 6° total vertex angle and a pipe with sudden expansion of the cross-section. Special care was taken to use suitable definitions of the diffuser efficiency and to obtain physically precise values for the inlet velocity distribution and the related boundary layer parameters. To influence the inlet boundary layer, straight pipes of various lengths were inserted between the air supply and the diffuser models. A wide range of inlet velocity profiles was thus obtained.—(1.5.1.1).

Theoretische und experimentelle Untersuchungen an Querstromgebläsen. R. Coester. Inst. fur Aero. Zurich. Mitt. 28. 1959. (*In German*).—(1.5.3.1).

LOADS—*see also* TESTING AND INSTRUMENTS

Base pressure at subsonic speeds in the presence of a supersonic jet. A. H. Craven. C.o.A. Report 129. *March* 1960.
The results of an experimental investigation into the effect of supersonic jets upon the base pressure of a bluff cylinder in a uniform subsonic flow are presented. The ratio of jet diameter to base diameter was 0·1875. A simple theory, based on a momentum integral, shows the dependence of the base drag upon the jet and free stream speeds and upon the dimensions of the jet and the base.—(1.6.1×1.4.1×27.1).

Interference effects due to vortex sheets according to slender body theory. A. von Baranoff. R.A.E. Lib. Trans. 892. *April* 1960.
Slender body theory is used to derive generalised formulae for the forces and moments exerted on the rear part of a missile by vortex sheets formed at a wing. Numerical examples show how these formulae may be applied.—(1.6.1×1.4.3×1.8.0.2).

The effects of thrust reversal at Mach numbers up to 0·86 *on the longitudinal and buffeting characteristics of a typical jet-transport airplane configuration. F. B. Sutton and J. J. Brownson. N.A.S.A. T.N. D*-136. *March* 1960.
Wind tunnel tests were conducted in the Ames 12 ft. pressure wind tunnel through ranges of angles of attack and jet-pressure ratios for forward and reverse thrust at Mach numbers from 0·40 to 0·86.—(1.6.3×1.8.2.2×1.3.4×27.1.1×5).

Power spectral analysis of some airplane response quantities obtained during operational training missions of a fighter airplane. H. A. Hamer and J. P. Mayer. N.A.S.A. T.N. D-366. *March* 1960.
Power spectra of normal and transverse load factor and pitching acceleration are presented for several types of missions normally performed by the aeroplane. The frequency content, which is described by the spectrum, provides information which is useful in control-system design and in the design of recording and computing equipment for analysing manoeuvre-load data.—(1.6.2×3.1×33.1.2).

PERFORMANCE

An optimization study of effects on aircraft performance of various forms of heat addition. B. S. Baldwin. N.A.S.A. T.N. D-74. *March* 1960.
Basic ram-jet aircraft design considerations are reviewed at a level of simplification appropriate for evaluation of external heat addition schemes. Similarity parameters are derived which allow comparison on a single plot of configurations with a wide range of design parameters.—(1.7×1.9.3×27.4).

STABILITY AND CONTROL—*see also* LOADS
WINGS AND AEROFOILS

Experimental investigation at a Mach number of 3.11 *of the lift, drag, and pitching-moment characteristics of five blunt lifting bodies. W. Letko. N.A.S.A. T.N. D*-226. *April* 1960.
The Reynolds number of the tests was 16×10^6 per foot. All the bodies tested were statically unstable about the geometric centre of the bodies, except one which was neutrally stable.—(1.8.2.2×25.2).

Statistical data on control motions and airplane response of a Republic F-84F. airplane during operational training missions. H. A. Hamer and J. P. Mayer. N.A.S.A. T.N. D-386. *March* 1960.
Results of a flight investigation conducted on a Republic F-84F service aeroplane during routine squadron operations are presented in the form of envelopes of maximum measured values and in terms of probability distributions of peak values. The data are presented according to type of mission and show to what extent the various missions performed by the aeroplane affected the magnitude and shape of the envelope plots and probability distributions.—(1.8.0.1×3.6×6.1).

THERMO-AERODYNAMICS—*see also* BOUNDARY LAYER
PERFORMANCE

Heat transfer to cylinders in crossflow in hypersonic rarefied gas streams. R. N. Weltmann and P. W. Kuhns. N.A.S.A. T.N. D-267. *March* 1960.
Heat transfer rates and temperature recovery ratios were measured on cylinders in crossflow in a low-density wind tunnel for three gases of different molecular weight and accommodation coefficient at hypersonic Mach numbers. The accommodation coefficient was 0·9 for nitrogen, 1·0 for argon, and 0·4 for helium. The data extend from free-molecule to almost continuum flow and cover ranges of Knudsen, Nusselt, and Reynolds numbers of $0{\cdot}001\leq Kn\leq15$, $0{\cdot}01\leq Nu\leq10$, and $0{\cdot}1\leq Re\leq1{,}000$.—(1.9.1×34·3×1.12.6).

Numerical computations of aerodynamic heating of liquid propellants. J. L. Kramer et al. *N.A.S.A. T.N. D-273. April* 1960.
A detailed numerical procedure, with an accompanying flow diagram, is presented for a digital computer programme to compute the aerodynamic heat input to liquid rocket propellents. The effect of a finite heat transfer coefficient between the liquid propellent and the tank wall is taken into account. Illustrative examples are presented.—(1.9.1×34.3×27.3).

Investigation of aerodynamic effects of external combustion below flat-plate model in 10- *by* 10-*foot wind tunnel at Mach* 2.4. *R. G. Dorsch* et al. *N.A.S.A. T.N. D-282. April* 1960.
Heat was added directly to the supersonic airstream by external combustion of aluminium borohydride. The pressure data are compared with previously reported small-tunnel results to obtain an appraisal of possible tunnel effects encountered in the small-tunnel tests.—(1.9.3×27.0×34.1.1.).

A theoretical study of stagnation-point ablation. L. Roberts. N.A.S.A. T.R. R-9. 1959.
A simplified analysis is made of ablation cooling near the stagnation point of a two-dimensional or axisymmetric body which occurs as the body vaporises directly from the solid state. The automatic shielding mechanism is discussed and the important thermal properties required by a good ablation material are given. The results of the analysis are given in terms of dimensionless parameters.—(1.9.1×1.1.1.4).

Effects of Mach number and wall-temperature ratio on turbulent heat transfer at Mach numbers from 3 *to* 5. *T. Tendeland. N.A.S.A. T.R. R-16.* 1959.
Heat-transfer data were evaluated from temperature time histories measured on a cooled cone-cylinder model with turbulent flow at Mach numbers 3·00, 3·44, 4·08, 4·56 and 5·04. The effects of Mach number and wall temperature ratio on Stanton number are compared with predictions from existing theories. The data were correlated by the T′ method used to correlate turbulent skin-friction data.—(1.9.1×1.1.3.4).

WINGS AND AEROFOILS—*see also* FLUID DYNAMICS

Forces et moments non linéaires induits par une aile avant sur une aile arrière et le fuselage, dans une configuration élancée. A. von Baranoff. O.N.E.R.A. N.T. 58. 1960. (*In French*).
Theoretical results are presented concerning the effects of a vortex sheet issuing from a forward wing and acting upon the afterbody of a slender configuration. Details of numerical application of the method are given. This application assumes that the aerodynamic properties of the forward part of the configuration, in particular the intensity of the vortex sheet immediately downstream of the trailing edge, are known. This information is usually readily obtained from slender body theory. It seems reasonable to use the results of linearised supersonic theory to take account of the effect of Mach number.—(1.10.1.2×1.2.2.1×1.4×1.8.0.2).

TESTING AND INSTRUMENTS—*see also* COMPRESSIBLE FLOW
THERMO-AERODYNAMICS
AEROELASTICITY

A method of forming continuous empirical equations for the thermodynamic properties of air from ambient temperatures to 15,000°*K, with applications. M. Grabau. A.E.D.C.-T.N.-59-*102. *Aug.* 1959.
This work relates generally to automatic data reduction for arc-driven, hotshot-type wind tunnels which operate at high stagnation temperatures. Pressure and density are used as the simplest independent variables. The thermodynamic functions for air, plotted at constant density as a function of the logarithm of the pressure, were found to exhibit basically similar graphical type-forms throughout the thermodynamic domain between 200°K. and 15,000°K. and densities between 10^{-6} and 10^{2} relative atmospheres.—(1.12.1.3×32.2.2).

Starting loads in an intermittent supersonic wind tunnel. S. L. Kinsolving and R. Jackson. Astia Doc. A.D.-208691. Jan. 1959.
Starting loads on an AGARD B Model were measured in a 12 in., variable-density, intermittent wind tunnel at Mach numbers of 2.0, 3.0 and 4.5. Two analytical methods are shown to approximate satisfactorily the maximum measured starting loads.—(1.12.1.3×1.6.1).

AEROELASTICITY

Flutter investigation of a true-speed dynamic model with various tip-tank configurations. J. L. Sewall et al. *N.A.S.A. T.N. D-178. March* 1960.
A 1/6 scale wing tip-tank model, representative of an unswept-wing fighter aeroplane, was flutter tested in the Langley 16 ft. transonic tunnel. The wing was of spar-balsa segment-type construction and was dynamically scaled to flutter at the same speed as a typical full-scale configuration. Each tip-tank housed a device for arresting flutter by providing for a quick shift in the tip-tank centre of gravity.—(2×1.12.1.2).

Structural damping in dynamic stability testing. C. J. Welsh and L. K. Ward. Astia Doc. A.D.-208776. Feb. 1959.
The results of a short study of structural-damping characteristics of cross-flexure pivots typical of those used in wind tunnel, dynamic-stability tests are presented. Although it has been usually assumed that the coefficient of the structural-damping term attributed to a cross-flexure pivot is constant, the experimental data here presented indicates that it varies approximately inversely with the frequency. It is shown that the structural-damped motion can be approximated adequately with the viscous-damping equation.—(2×1.12.1×33.1.2).

AIRCRAFT

See AERODYNAMICS—LOADS
STABILITY AND CONTROL

DESIGN AND CONSTRUCTION

See STRUCTURES—THEORY AND ANALYSIS

FATIGUE

Rolling-contact fatigue life of a crystallized glass ceramic. T. L. Carter and E. V. Zaretsky. N.A.S.A. T.N. D-259. March 1960.
The rolling-contact fatigue properties of a crystallised glass ceramic were investigated with the five ball fatigue tester. The effects of contact stress, contact angle, test lubricant, and test temperature up to 700°F. are reported.—(31.2.2.9.2.1×21.3.1).

Effect of friction on the fatigue strength of single row riveted double cover plate joints in clad 2024-*T aluminum alloy. A. Hartman and F. A. Jacobs. N.L.L.-T.M. M.* 2064. *Nov.* 1959.
The mean fatigue strength at repeated tension for an endurance of 107 load cycles has been determined of double cover plate specimens in clad 2024-T aluminium alloy sheet with one row of 2117-T aluminium alloy snap rivets. The fatigue strength was determined by the staircase method. A comparison of these fatigue strengths has been used to obtain an estimate of the load transfer by friction between the sheets.—(31.2.2.3.2.1×31.2.3.2.2.2×21.2.2).

SCIENCE—GENERAL

See also AERODYNAMICS—COMPRESSIBLE FLOW
FLUID DYNAMICS
TESTING AND INSTRUMENTS

Mass spectrometer measurements of the ion composition of the upper atmosphere from the third artificial earth satellite. V. G. Istomin. R.A.E. Lib. Trans. 875. *Feb.* 1960.—(32.2×24).

Fundamentals of acoustical silencers. (III) Attenuation characteristics studied by an electrical simulator. J. Igarashi and M. Arai. Aero. Res. Inst. Tokyo Report 351. *Feb.* 1960.
The method of measuring the attenuation characteristics by the electric simulator is presented. Several acoustic elements were simulated by the electric elements. Without constructing any real models of the silencers the attenuation characteristics can be obtained by proper selection of the electric elements. Effect of length of the pipe which connects the engine and the muffler was also studied by this method.—(32.2.3).

STRUCTURES

LOADS—*see* AERODYNAMICS—LOADS
AEROELASTICITY
THEORY AND ANALYSIS

THEORY AND ANALYSIS

Contraction and internal stresses in bonded joints. W. Spaeth. R.A.E. Lib. Trans. 886. *March* 1960.
The shrinkage and post-shrinkage which occur in adhesives of

high polymer materials set up internal stresses in the bond. These stresses are also set up by the different expansion coefficients of adhesive and metal. The temperature variation of the mechanical properties is considered. From theoretical considerations some characteristic properties of bond joints are interpreted *e.g.* dependence of bond strength on temperature, the influence of the thickness of the adhesive.—(33.2.4.13).

Analysis of elastic-plastic stress distribution in thin-wall cylinders and spheres subjected to internal pressure and nuclear radiation heating. D. F. Johnson. N.A.S.A. T.N. D-271. April 1960.
A finite-difference analysis utilising a successive-approximation technique was made using the properties of Inconel X. Distributions of stress and strain for values of heat flux from 50 to 950 watts per cubic centimeter and pressure-times-radius values from 18,000 to 42,000 pounds per inch are presented. The existence of an optimum thickness and other problems unique in the design of a nuclear-reactor pressure vessel are discussed. —(33.2.4.3.7×33.2.4.3.9).

Natural frequencies in bending of twisted rotating and non-rotating blades. G. Isakson and J. G. Eisley. N.A.S.A. T.N. D-371. March 1960.
The effect of twist on the natural frequencies of uniform and tapered non-rotating blades and the effect of twist and blade angle on the natural frequencies of rotating uniform blades are shown by means of charts. The Raleigh-Southwell procedure for determining the effect of rotation on natural frequencies of beam vibration is evaluated, and the Lo and Renbarger relation for the effect of blade angle on the natural frequencies of a rotating blade is discussed.—(33.2.4.1.2×4.4.1).

The theory of momentless shells of revolution. V. Z. Vlasov. N.A.S.A. T.T. F-6. April 1960.
General homogeneous solutions are presented for the equations of the membrane theory of shells of revolution of both positive and negative Gaussian curvature. Solutions are obtained of a number of boundary value problems of edge-loaded shells and compound shells loaded at their junctures, including a compound shell made of conical segments with stiffeners at the junctures.—(33.2.4.11).

On solutions for the transient response of beams. R. W. Leonard. N.A.S.A. T.R. R-21. 1959.
Williams type modal solutions of the elementary and Timoshenko beam equations are presented for the response of several uniform beams to a general applied load. Example computations are shown for a free-free beam subject to various concentrated loads at its centre. Discussion includes factors influencing the convergence of modal solutions and factors to be considered in a choice of beam theory. Results obtained by two numerical procedures, the travelling-wave method and Houbolt's method, are also presented and discussed.—(33.2.4.1×33.1.2).

The stresses around reinforced elliptical holes in plane sheet. W. H. Wittrick. A.R.L. Report S.M. 267. May 1959.
An analytical solution is given for the problem of the stress distribution due to an elliptical hole, reinforced around its boundary, in a plane sheet subjected at infinity either to an arbitrary constant stress system or to a bending type stress system. Numerical results are presented in tables showing the variation of stress in the sheet around the boundary of the hole. The loading case corresponding to that arising in a pressurised fuselage, in which one principal stress at infinity is twice the other, is considered in detail.—(33.2.4.5.8×33.2.4.13.8).

The lateral stability of uniform elastic beams. J. Solvey. Aust. A.R.C. Report A.C.A.-60. May 1959.
The values of critical loads causing lateral buckling of uniform beams in the elastic range are presented. Two classes of beams have been considered: those having negligible warping-constants and those having finite warping-constants. The results provide an easy method of computing critical loads.—(33.2.4.1.6).

Thermal stressing due to an arbitrary heat source in a circular cut-out. G. R. Cowper. N.R.C. Report M.S.-101. March 1960.
Formulae for the thermal stresses and displacements in a circular elastic plate with a concentric circular cut-out due to an arbitrary but steady heat input at the edges of the plate are presented. Formulae for thermal stresses in an infinite plate with a single circular cut-out are obtained as a limiting case. Numerical results are worked out for the case in which an arbitrary arc of the edge of the cut-out is held at a high temperature while the remainder of the edge remains cool.—(33.2.4.5.9).

Organization of a large computation in aircraft stress analysis. A. L. M. Grzedzielski. N.R.C. Report L.R.-257. July 1959.
Theoretical and organisational aspects of a large scale numerical stress analysis covering effects of applied loads and uneven temperature are discussed. The underlying stress analytical theory and the required matrix symbolism are explained using the example of a swept multi-spar model box for which experimental data are available. A large scale computation, actually performed during the design of a proptotype aircraft, is described.—(33.2×22.1).

The effect of axial constraint on the instability of thin circular cylindrical shells under external pressure. J. Singer. Israel Inst. Tech. Dept. Aero. Eng. Report 2. Sept. 1959.
The effect of axial elastic restraint on the instability of a circular cylindrical shell under hydrostatic pressure, or under uniform lateral pressure, is analysed by a Rayleigh Ritz approach. For hydrostatic pressure the effect is calculated for a wide range of parameters and design curves are presented for the percentage increase in critical pressure.—(33.2.4.3.7).

Simplified equations for thin circular conical shells under arbitrary loads and arbitrary temperature distributions. J. Singer. Israel. Inst. Tech. Dept. Aero. Eng. Report 4. June 1959.
Simplified differential equations governing the deformations of deep thin circular conical shells subjected to arbitrary loads and arbitrary temperature distributions are derived by the principle of the minimum total potential energy. The corresponding boundary conditions are also obtained.—(33.2.4.11).

Buckling of conical shells under external pressure and thermal stresses. J. Singer and N. J. Hoff. Israel Inst. Tech. Dept. Aero. Eng. Report 6. Dec. 1959.
Simplified differential equations governing the deformations of deep thin circular conical shells subjected to arbitrary loads and arbitrary temperature distributions are derived by the principle of the minimum total potential energy.—(33.2.4.11.7×33.2.4.11.9)

Deformation and thermal stress of rectangular beams or flat strips heated at one surface. M. Uemura. Aero. Res. Inst. Tokyo Report 352. March 1960.
The problem of transient heating conditions in a simple thin beam having rectangular cross-section is considered. The two cases of clamped and simply supported ends where axial displacements are restrained are discussed, taking account of the finite deformation.—(33.2.4.1.9).

Weight Analysis and Control

Effects of design speed and normal acceleration on aircraft structure weight. M. E. Burt. C.P. 490. 1960.
Graphs show the structure weight of typical fighter and bomber aircraft for ranges of design diving speed and maximum normal acceleration. Variations of wing loading and geometry, and spar or box-beam wing construction are investigated.—(33.4.1).

THERMODYNAMICS

See also Aerodynamics—Thermo-Aerodynamics

A one-dimensional theory of liquid-fuel rocket combustion. III: the effect of non-uniform droplet radii, injection velocities and physical properties. J. Adler. C.P. 469. 1960.
The one-dimensional liquid-fuel rocket combustion theory of Spalding is extended to apply to models consisting of two different droplet groups. In one model, droplet radii and injection velocities are varied, and in the other the particle densities and mass transfer numbers are non-uniform. Solutions are obtained for the variation of droplet radius, velocity and gas velocity with distance from the injection end.—(34.1.1×27.3).

Experimental results of a heat-transfer study from a full-scale pebble-bed heater. R. B. Lancashire et al. *N.A.S.A. T.N. D-265. March* 1960.
Data were obtained from the initial operation of a large regenerative pebble-bed heat exchanger to determine the heat transfer characteristics of a packed bed of spheres. The facility, designed to produce high temperature air for hypersonic flight research, is described in detail. The heat transfer analysis and results are presented and compared with those of other investigators.—(34.3×27.1.2×27.4.2).